Common Core State Standards

for

Mathematics

Math Plus Academy Seminar

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Outline for Today

- Background
- A look inside the Common Core State Standards
- Do some math
- Suggestions for parents

Underlying Principle

- "*Everyone is good at mathematics because everyone can think. And mathematics is about thinking.*"
  - Yeap Ban Har, National Institute of Education, Singapore.

- Corollary 1: Strategies that attempt to remove thinking from learning are bound to fail in the long run.
- Corollary 2: When learning is effective, "getting the right answer" is but a small piece of the work.
- Corollary 3: The most important thing teachers and parents can do is convince students that their thinking matters.

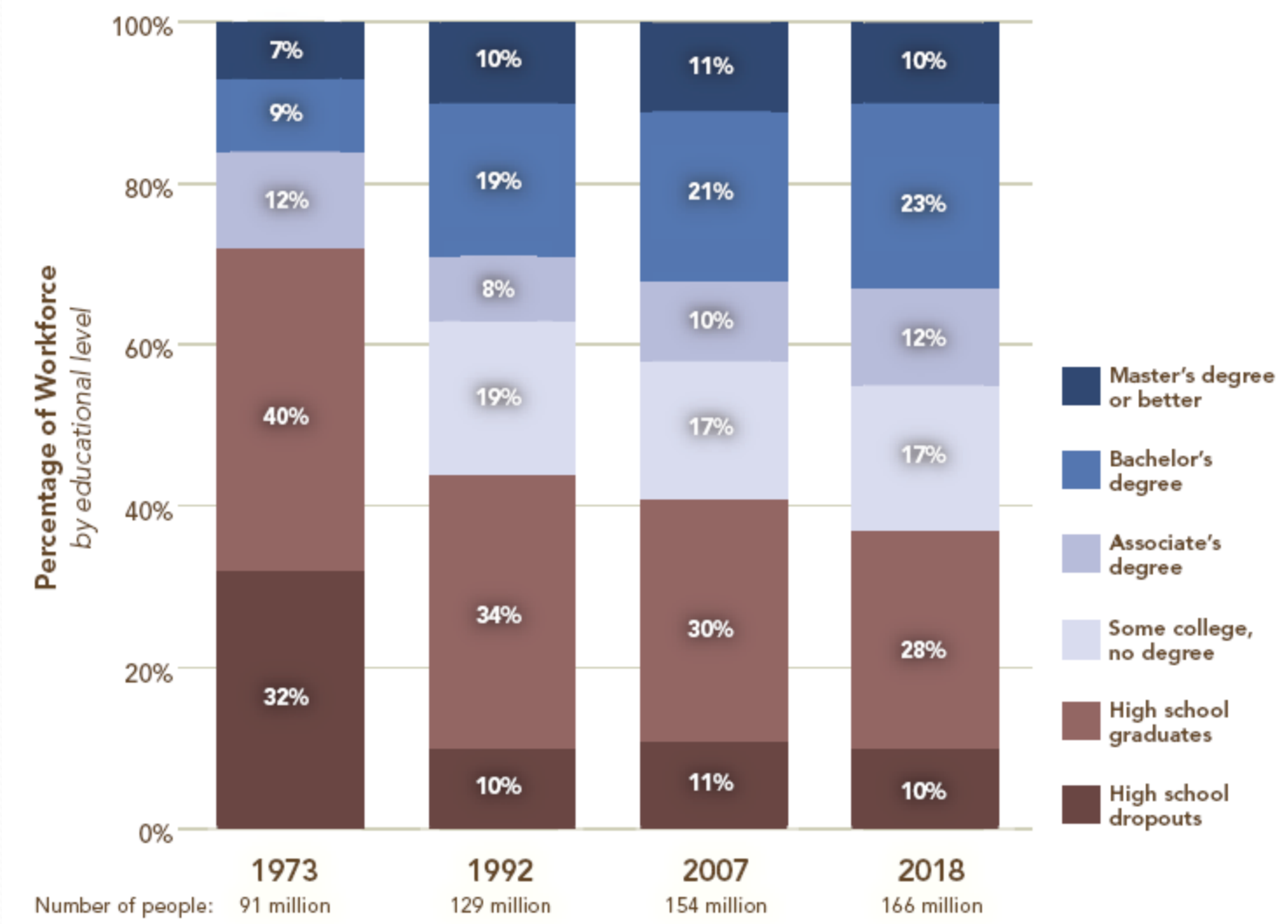
Background

on the Common Core State Standards (CCSS)

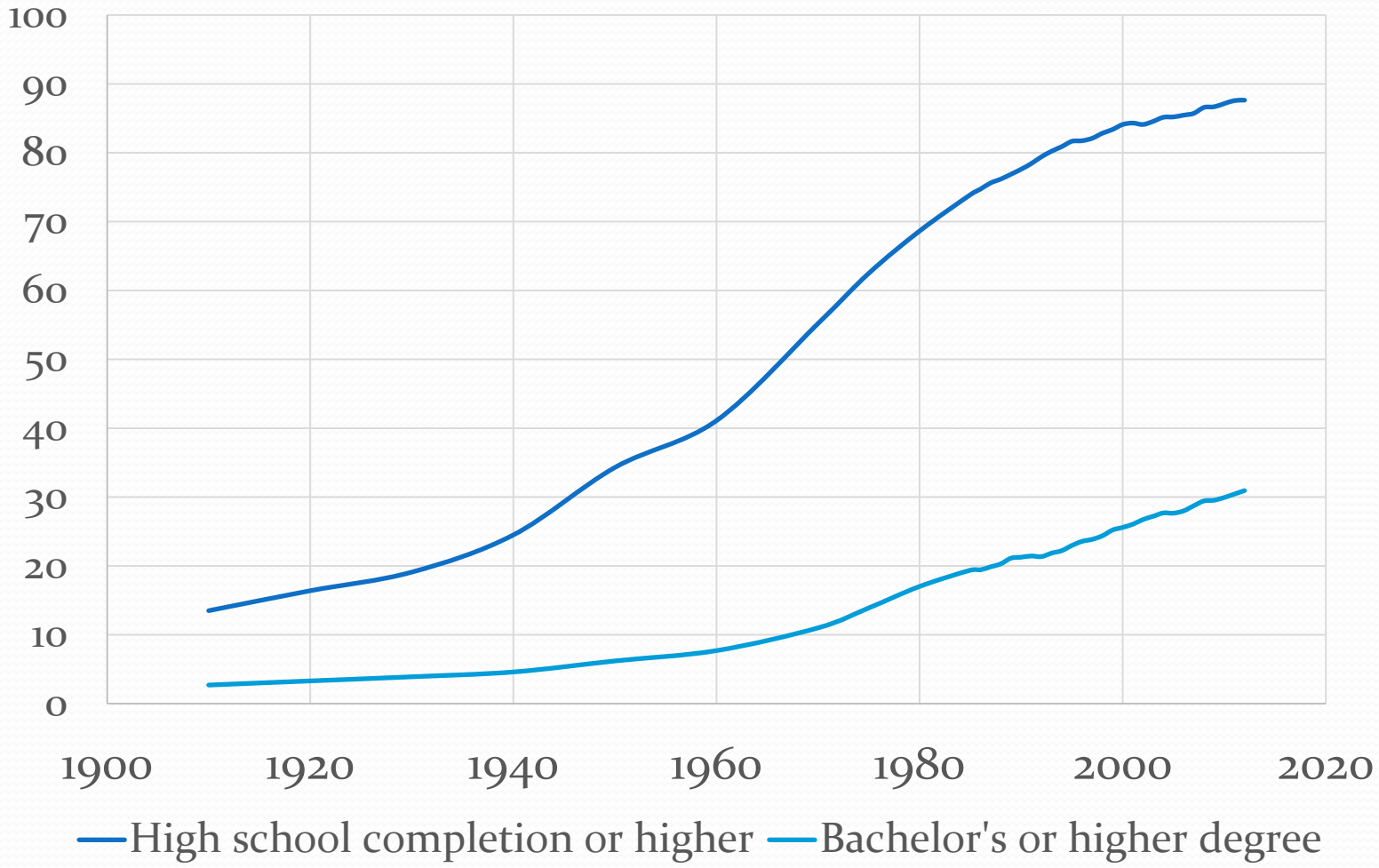
Problems the CCSS Aims to Solve

- Changing economy
- Increasing requirements in the workplace
- High college remediation rates
- Low college completion rates
- “Mile-wide, inch-deep” standards and curricula

More Jobs Require Some College



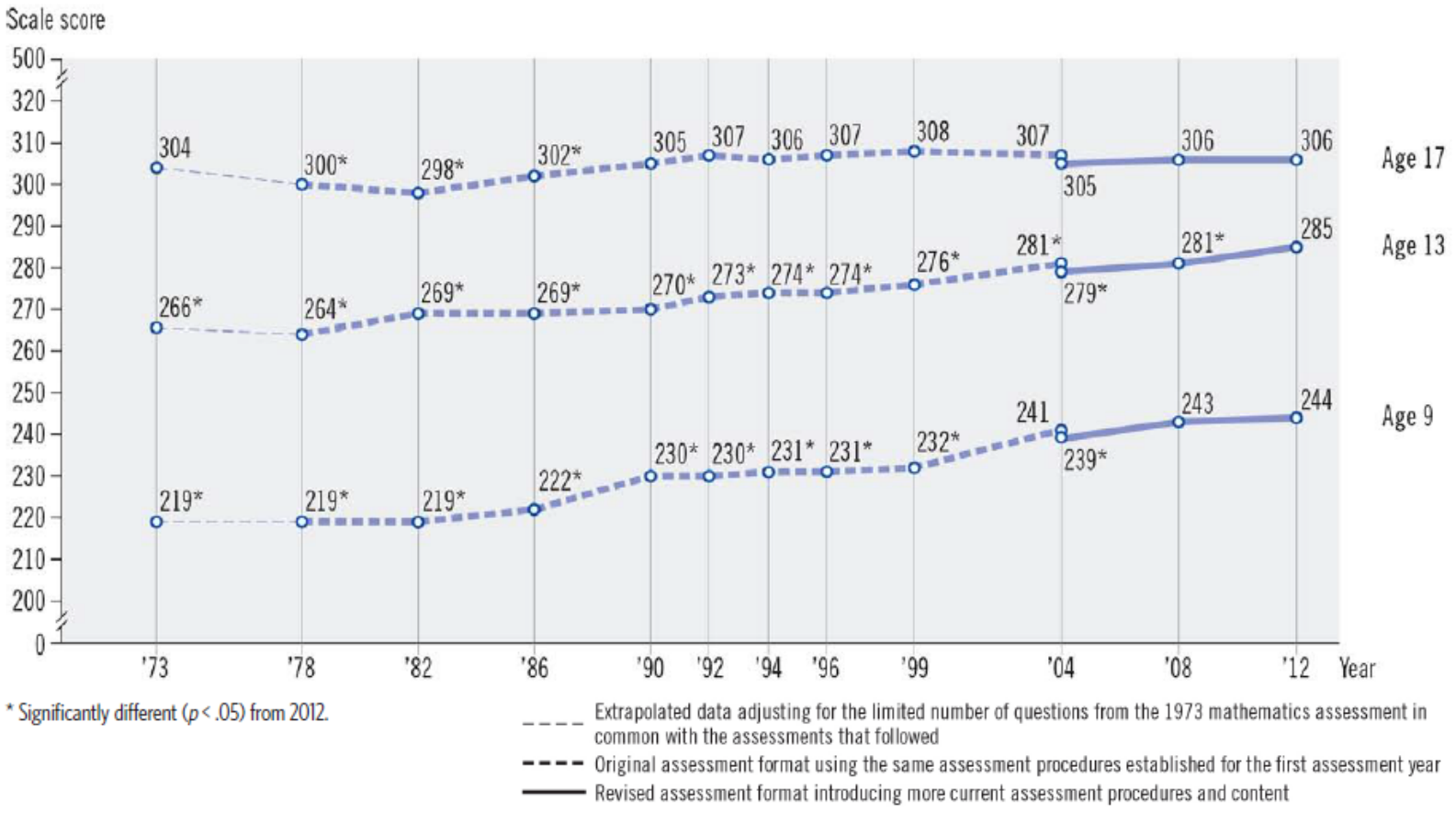
U.S. Educational Attainment, (% of Persons Age 25 and Over)



International Comparisons

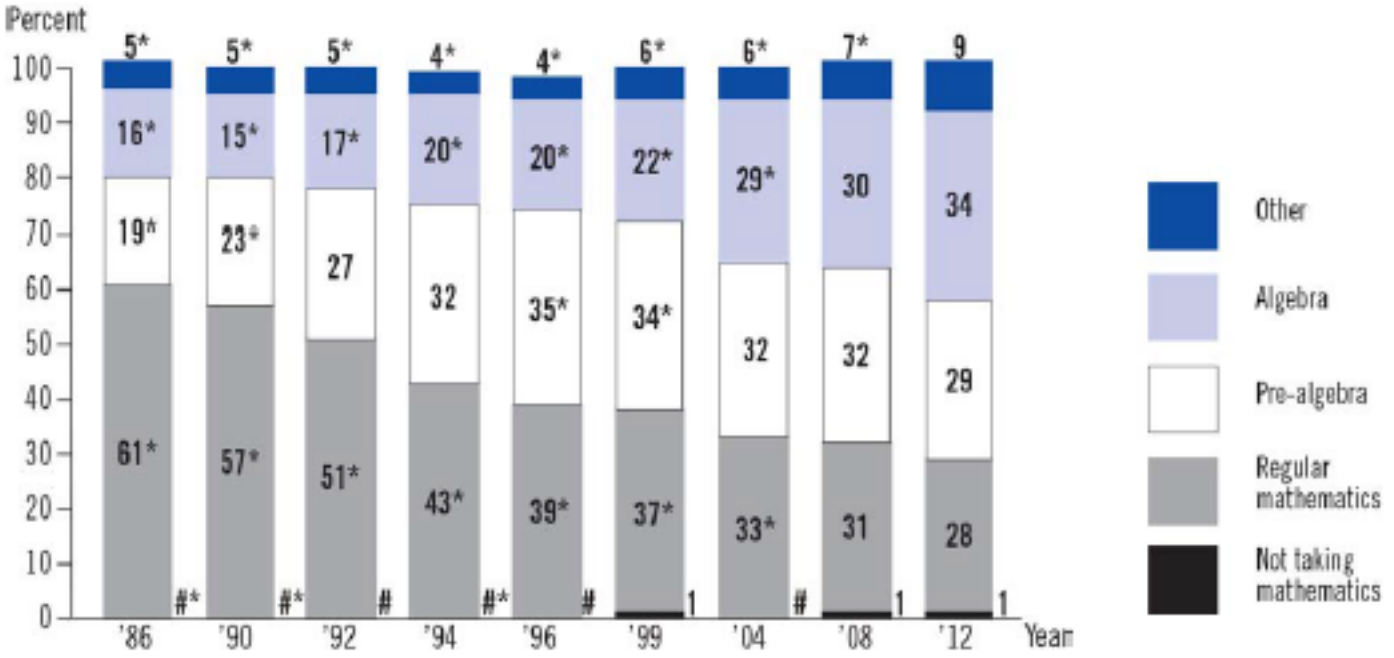
- U.S. high school and college completion show clear improvement
- … but slow growth rates have let the U.S. slip behind many other nations
- … and U.S. adults do not perform well compared to comparably educated adults in other countries

Mathematics Achievement Long-Term Trend (NAEP)



Math Course Taking, Age 13

Figure 33. Trend in percentage of 13-year-old students assessed in NAEP mathematics, by type of mathematics taken during the school year



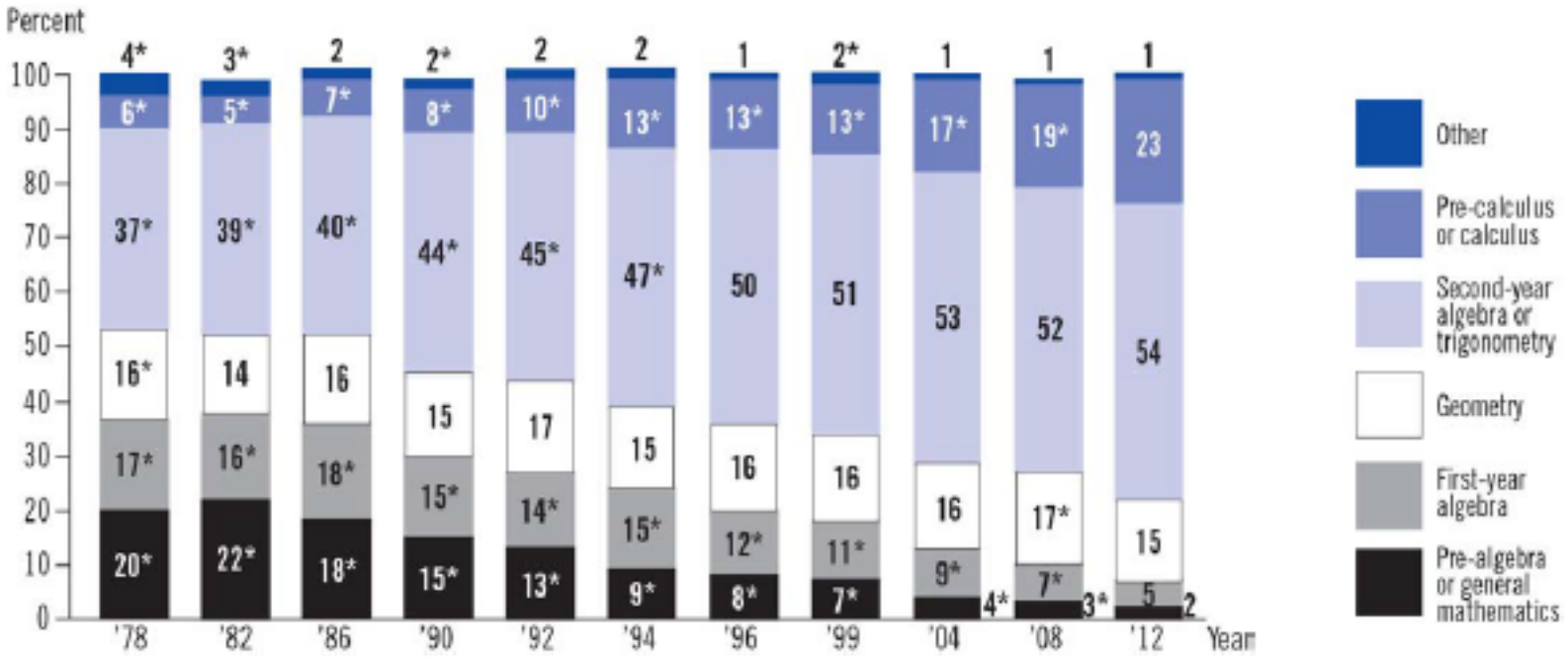
Rounds to zero.

* Significantly different ($p < .05$) from 2012.

NOTE: Results for 1986–2004 are from the original assessment format, and results for 2008 and 2012 are from the revised assessment format (2004 revised assessment format results are not available). Detail may not sum to totals because of rounding.

Math Course Taking, Age 17

Figure 35. Trend in percentage of 17-year-old students assessed in NAEP mathematics, by highest level mathematics course ever taken



* Significantly different ($p < .05$) from 2012.

NOTE: The "pre-algebra or general mathematics" response category includes "pre-algebra or introduction to algebra," "general, business, or consumer mathematics," and students who did not take any of the listed courses. The "other" response category includes students for whom the highest level mathematics course could not be determined due to missing or inconsistent responses. Results for 1978–2004 are from the original assessment format, and results for 2008 and 2012 are from the revised assessment format (2004 revised assessment format results are not available). Detail may not sum to totals because of rounding.

Remediation Rates (Percent, 2004 high school seniors)

	4-year institution			2-year institution		
	Remedial reading	**Remedial writing**	**Remedial math**	**Remedial reading**	**Remedial writing**	**Remedial math**
Overall	16.2	24.8	25.8	26.9	29.8	38.7
By highest math course taken						
Pre-algebra or below	25.1	31.1	38.2	41.3	39.0	49.5
Algebra I/geometry	17.0	23.5	41.3	30.6	34.6	44.1
Algebra II	21.2	30.9	36.2	28.0	29.6	40.8
Trig./analytic geom./stat.	15.9	25.1	25.2	25.4	29.1	34.2
Pre-calculus/calculus	13.9	22.4	20.4	17.7	21.1	25.2

Achievement Trends

- Achievement is up by many indicators
  - Significant growth in grades 4 and 8
  - High school diploma, math course taking
  - College attendance, college completion
- High school achievement is flat
  - U.S. 15-year-olds lag in applying math
  - Poor results on H.S. end-of-course exams
  - College remediation rates remain high
- **Today's world demands more**

College and Career Readiness

- Algebra 1 and Geometry provide insufficient readiness for college and most careers
- More states are requiring Algebra 2 or its equivalent (A2E)
- All students need proficiency in A2E for
  - Many careers, with or without college
  - Informed citizenship
  - Individual empowerment
- High school mathematics should open doors
  - But adult decisions often close doors for students
  - After students complete A2E, they have choices

List of Acronyms (LOA)

- Ohio Department of Education (ODE)
- Council of Chief State School Officers (CCSSO)
- National Governors Association (NGA)
- English Language Arts (ELA)

CCSSM Development Timeline

- 2007: ODE and the State Board began discussing standards revision
- 2007: CCSSO discussed the possibility of common standards in math and ELA
- 2009: NGA joined the Common Core State Standards Initiative
- Summer, 2009: ODE convened standards revision committees
- Summer, 2009: 48 states chose to participate in the development of common standards
- November, 2009: Common Core writing teams announced
  - Ohio's work was input for this process
- March, 2010: Public drafts released for comment
- June, 2010: Final version released
- 2010-2011: about 45 states adopted these standards

Overview of the CCSSM

Common Core State Standards for Mathematics

CCSS Principles

- **Focus**: focus strongly on key ideas, understandings, and skills in each grade and course
- **Coherence**: think across grades, and link to major topics in each grade
- **Rigor**: in major topics, pursue with equal intensity
  - conceptual understanding,
  - procedural skill and fluency, and
  - applications

CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Grade Level Overview

Mathematics | Grade 2

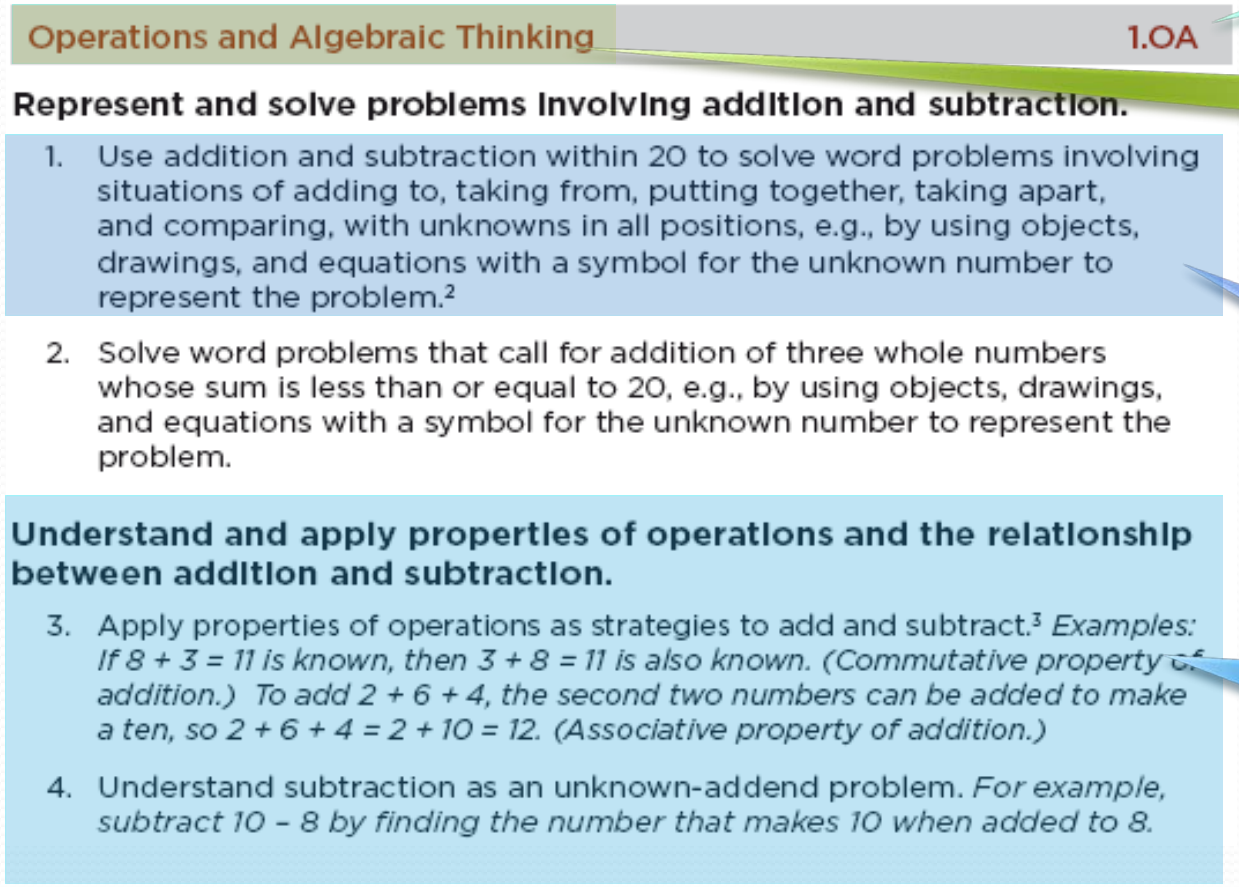
In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. T
includes ideas of counting in fives, tens, and multiples of hundreds, t
and ones, as well as number relationships involving these units, in
comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000

Critical Area of Focus

Format of K-8 Standards



Grade Level

Domain

Standard

Cluster

K-8 CCSS Changing Content Emphases

- Primary focus on number in grades K-5
- Fractions as numbers on the number line, beginning with unit fractions
- Fluency with standard algorithms, supported by strategies based in place value
- Much statistics in grade 6-8
- Much algebra and geometry in grades 7-8

Key Ideas Flower in the Middle Grades

- Solving problems with rational numbers
- Proportional relationships, unit rates, and percentages
- Area, surface area, and volume
- Linear equations and functions

- *Continue to use all of these **often** in high school, college, careers,*

Understanding the Standards

Learning Addition Facts (Grade 1)

- 1.OA.6. Add and subtract within 20, demonstrating fluency for addition [illegible] within 10. Use strategies s[illegible] (e.g., 8 + 6 = 8 + 2 + [illegible]mber leading [illegible] = 9); using th[illegible]d subtract[illegible]12, one knows 12 – 8 = 4); an[illegible]ent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

How do we read this?
Not like we read a novel.

Learning Addition Facts (Grade 1)

- 1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.
- Use strategies such as
  - counting on;
  - making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
  - decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9);
  - using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4);
  - and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Mathematical Thinking Strategies

- Although these sound like teaching strategies, they are powerful mathematical thinking strategies:
  - Making ten and decomposing to ten help students deepen their understanding of place-value notation
  - A related "make one hundred" strategy is useful in thinking about change from a dollar
  - The relationship between addition and subtraction helps students with algebra
- These strategies promote flexibility and support connected learning of more than 200 arithmetic facts
  - … which is a lot, if students rely on memory alone.

Learning Addition Facts (Grade 2)

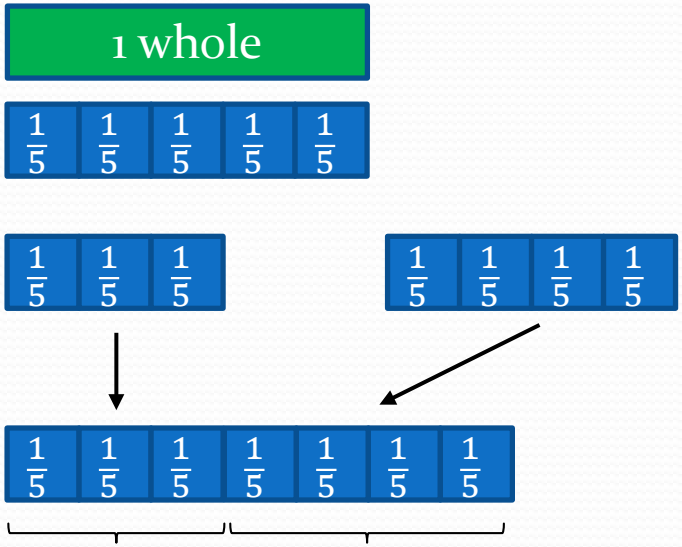
- 2.OA.2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
- Implications:
  - Students are still required to know the facts from memory
  - With the Common Core, students can fall back on thinking strategies whenever memory fails
  - Students can use those same thinking strategies to support other mathematics learning

The Fraction Foundation (Grade 3)

- 3.NF.1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
- Some folks respond, “I have no idea what that means.” “Too many letters.”

- Here is a specific version:
- 3.NF.1. Understand a fraction 1/5 as the quantity formed by 1 part when a whole is partitioned into 5 equal parts; understand a fraction 3/5 as the quantity formed by 3 parts of size 1/5.

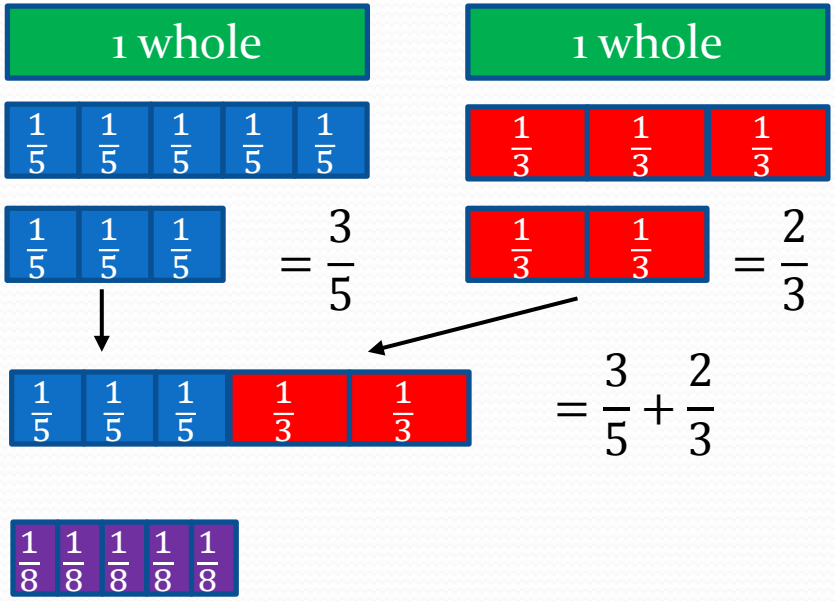
What is $\frac{3}{5} + \frac{4}{5}$?



$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$$

- Focus on meaning:
  - 3 parts of size $\frac{1}{5}$ plus 4 parts of size $\frac{1}{5}$
- From arithmetic of whole numbers, the answer is 7 parts of size $\frac{1}{5}$
- Improper fractions are very useful
  - Not to be discouraged
- Subtraction follows similarly

What is $\frac{3}{5} + \frac{2}{3}$?



- Can we proceed the same way?
- The answer is the combined rectangles, but …
- We can see the sum > 1
- We can see why some other answers are wrong
- We need the same size parts!

The Square and Triangle Problem

□ + □ + □ + △ = 47

□ – △ = 1

- From the second equation, I know that square is one more than triangle.
- Then I imagined that the triangle in the first equation was a square.
- Then the sum would be one more, so the 4 squares would equal 48.
- The square is 12 and the triangle is 11.

□ = △ + 1

Add the two equations …

Divide by 4 …

Chicken and Egg Problems

33

A Rich Problem

- A hen-and-a-half lays an egg-and-a-half in a day-and-a-half. How many eggs would 6 hens lay in 4 days?

  - Think
  - Draw a picture
  - Make a table

Extension Questions

a) 6 hens, 4 days. How many eggs?

b) 8 hens, 16 eggs. How many days?

c) 12 eggs, 3 days. How many hens?

d) 12 eggs, 7 days. How many hens?

A Pictorial Solution

6 hens

Hh Hh Hh Hh

Ee Ee Ee Ee 1.5 days

Ee Ee Ee Ee 1.5 days

So 6 hens lay 12 eggs in 3 days

We need one more day

A Pictorial Solution

6 hens

Hh Hh Hh Hh

Ee	Ee	Ee	Ee	1.5 days
Ee	Ee	Ee	Ee	1.5 days
E	E	E	E	1 day

So 6 hens lay 16 eggs in 4 days

A Tabular Solution

Hens	Days	Eggs
1.5	1.5	1.5
3	1.5	3
3	3	6
3	1	2
6	1	4
6	4	16

Let’s start again

We are beginning a new unit in ninth grade

Chicken and Egg Problems

- A hen-and-a-half lays an egg-and-a-half in a day-and-a-half. How many eggs would 6 hens lay in 4 days?

Use the Units

$$\left(\tfrac{3}{2}\,\text{egg}\right)/\left(\tfrac{3}{2}\,\text{hen}\right)/\left(\tfrac{3}{2}\,\text{day}\right) = \left(1\ \text{egg/hen}\right)/\left(\tfrac{3}{2}\,\text{day}\right)$$

$$= \frac{2}{3}\,\text{egg}/\text{hen}/\text{day}$$

$$= \frac{2}{3}\,\frac{\text{egg}}{\text{hen}\cdot\text{day}}$$

6 hens, 4 days. How many eggs?

$$\frac{2}{3}\,\frac{\text{egg}}{\text{hen}\cdot\text{day}}\cdot 6\,\text{hens}\cdot 4\,\text{days} = 16\,\text{eggs}$$

- The homework is 1-49 odd. You may get started now.

* ***Stop*** *

Question

- What is a hen-day?

Other questions

b) 8 hens, 16 eggs. How many days?

$$\frac{16\,\text{eggs}}{8\,\text{hens}} \cdot \frac{3}{2} \frac{\text{hen} \cdot \text{day}}{\text{egg}} = 3\,\text{days}$$

c) 12 eggs, 3 days. How many hens?

$$\frac{12\,\text{eggs}}{3\,\text{days}} \cdot \frac{3}{2} \frac{\text{hen} \cdot \text{day}}{\text{egg}} = 6\,\text{hens}$$

d) 12 eggs, 7 days. How many hens?

$$\frac{12\,\text{eggs}}{7\,\text{days}} \cdot \frac{3}{2} \frac{\text{hen} \cdot \text{day}}{\text{egg}} = \frac{18}{7}\,\text{hens}$$

About Proportional Reasoning

- When the emphasis is “set up a proportion and cross multiply”
  - Many students learn only procedures
  - Many students “cross multiply” for fraction addition, subtraction, multiplication, or division
- Instead, let’s emphasize
  - Proportional relationships are about quantities that vary together (like hens and eggs)
  - Write an equation and solve

Suggestions for Parents

- Praise effort and perseverance
  - Fixed versus growth mindsets
  - Value “productive struggle”
- Ask them to explain their thinking and reasoning
  - … especially when they are correct
  - How did you get that? How do you know?
  - Let them do the thinking
  - Let them keep the pencil
- Support your children’s teachers
- See the resources from the National PTA and the Ohio PTA

Questions?