HS Pathways in the CCSS for Mathematics

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CCSSM and Acceleration

- The CCSS for Mathematics represent significant curricular acceleration in grades K-8
  - Much of Algebra 1 and Geometry are in the middle grades
  - Many “accelerated” programs will no longer be ahead
  - The CCSS for Grade 8 is a reasonable, internationally benchmarked response to Algebra for all in grade 8
- Accelerating large percentages of students much beyond the CCSS for K-8 is probably unwise
- The CCSSM for high school include much advanced content and much new content for all students
- *So we need to rethink mathematics, grades 7-12*

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Underlying Principle

- "*Everyone is good at mathematics because everyone can think. And mathematics is about thinking.*"
  - Yeap Ban Har, National Institute of Education, Singapore.
- Corollary 1: Strategies that attempt to remove thinking from learning are bound to fail in the long run.
- Corollary 2: When learning is effective, "getting the right answer" is but a small piece of the work.

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Overview

- Response to Intervention (RtI)
- A look inside the system
- Toward college and career readiness
- What is needed?
- A look inside the CCSS for High School mathematics
- A look inside the Model Pathways
- About serving all students
- Implementation suggestions and resources

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What Is RtI?

- RtI is about establishing a school-wide system for allocating instructional resources where they are needed
  - Give all students access to the regular curriculum AND provide differentiated instruction and support
  - Some students are 15 minutes behind; others are years behind
  - Labels are less important than providing additional instruction where it is needed
  - RtI integrates regular and special education

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What Is Not RtI?

- RtI is not a package
- RtI is neither tracking nor homogeneous grouping
  - RtI is *not* about providing different instruction to different groups of students, based on adult judgments about what students cannot do
- When it comes to mathematical thinking, any group of 2 or more students is heterogeneous
- And perhaps you have encountered students who seemed to be heterogeneous all by themselves

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RtI Is About Instruction

- RtI requires high-quality, differentiated Tier 1 instruction
  - Differentiation happens within RtI, in each Tier
  - Teachers should be differentiating their instruction all the time based on insights into student thinking
  - If large percentages of students seem to need Tier 2 support, examine regular instruction
  - Many learning difficulties are caused by poor instruction

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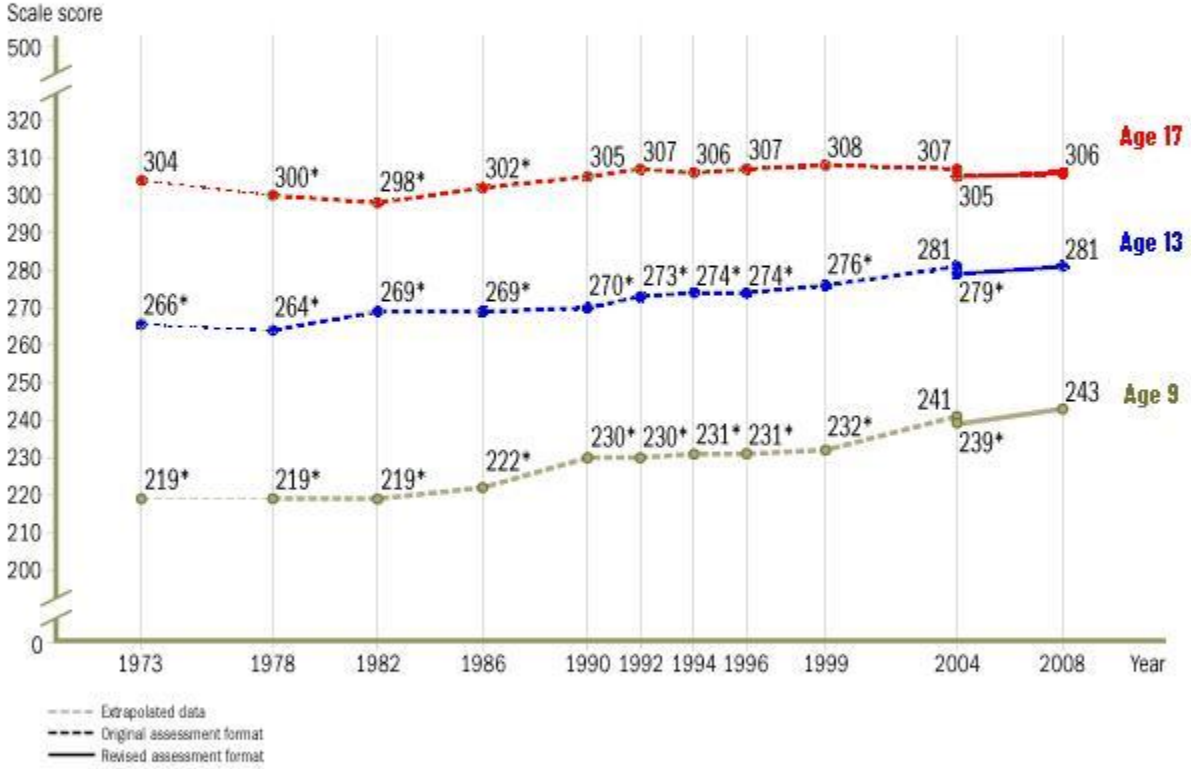
Trends in National Average Mathematics Scores



Since 1973	Since 2004
↔	↔
↑ 15 points	↑ 3 points
↑ 24 points	↑ 4 points

↑ Indicates the score was higher in 2008.
↔ Indicates there was no significant change in the score in 2008.

Mathematics Achievement Trends

- Achievement is up by many indicators
  - Significant growth in grades 4 and 8
  - High school diploma, math course taking
  - College attendance, college completion
- High school achievement is flat
  - U.S. 15-year-olds lag in applying math
  - Poor results on H.S. end-of-course exams
  - College remediation rates remain high
- **Today's world demands more**

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Stagnancy Is not a Crisis!

- A crisis is a “turning point”
  - John Ewing, Math for America
- We have long-term structural problems
  - And many inadequate improvement initiatives
- We need to make fundamental long-term changes over decades

- The Common Core State Standards are a response to this challenge

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Toward College and Career Readiness

- More states are requiring Algebra 2 or its equivalent (A2E)
  - A proxy for college and career readiness
- CCSS provides a definition of college and career readiness: All standards not indicated by (+)
- We need to make A2E rigorous, relevant, and attainable
  - Your parents' Algebra 2 will not do
  - Is this feasible?
- What is the rationale?

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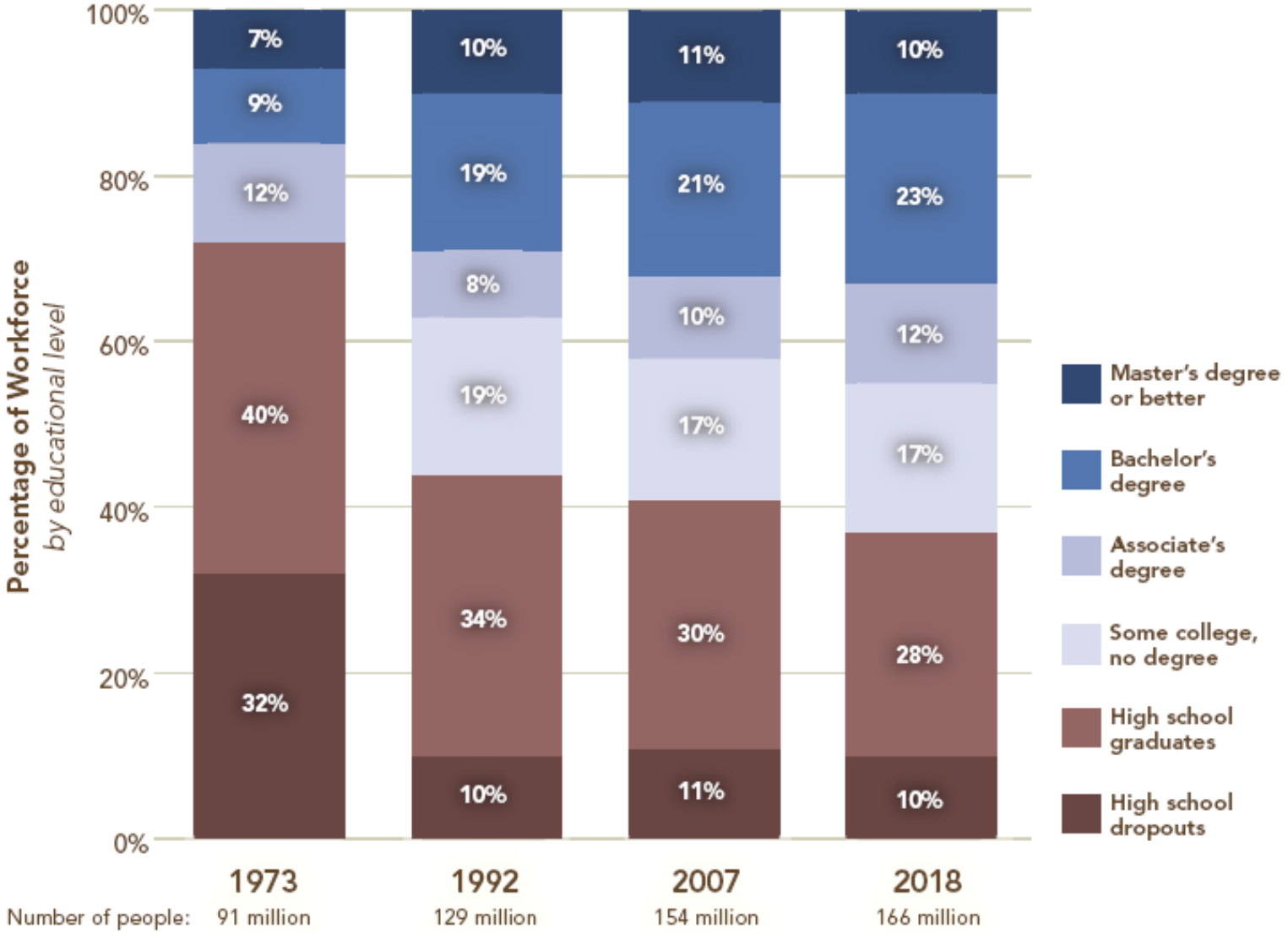
More Jobs Require Some College



Not Enough College Graduates

- “Demand for workers with college educations will outpace supply to the tune of 300,000 per year. By 2018, the postsecondary system will have produced 3 million fewer college graduates than demanded by the labor market.”

(Carnevale, Smith, Strohl, 2010, *Help Wanted*, p. 16)

See http://cew.georgetown.edu/jobs2018/

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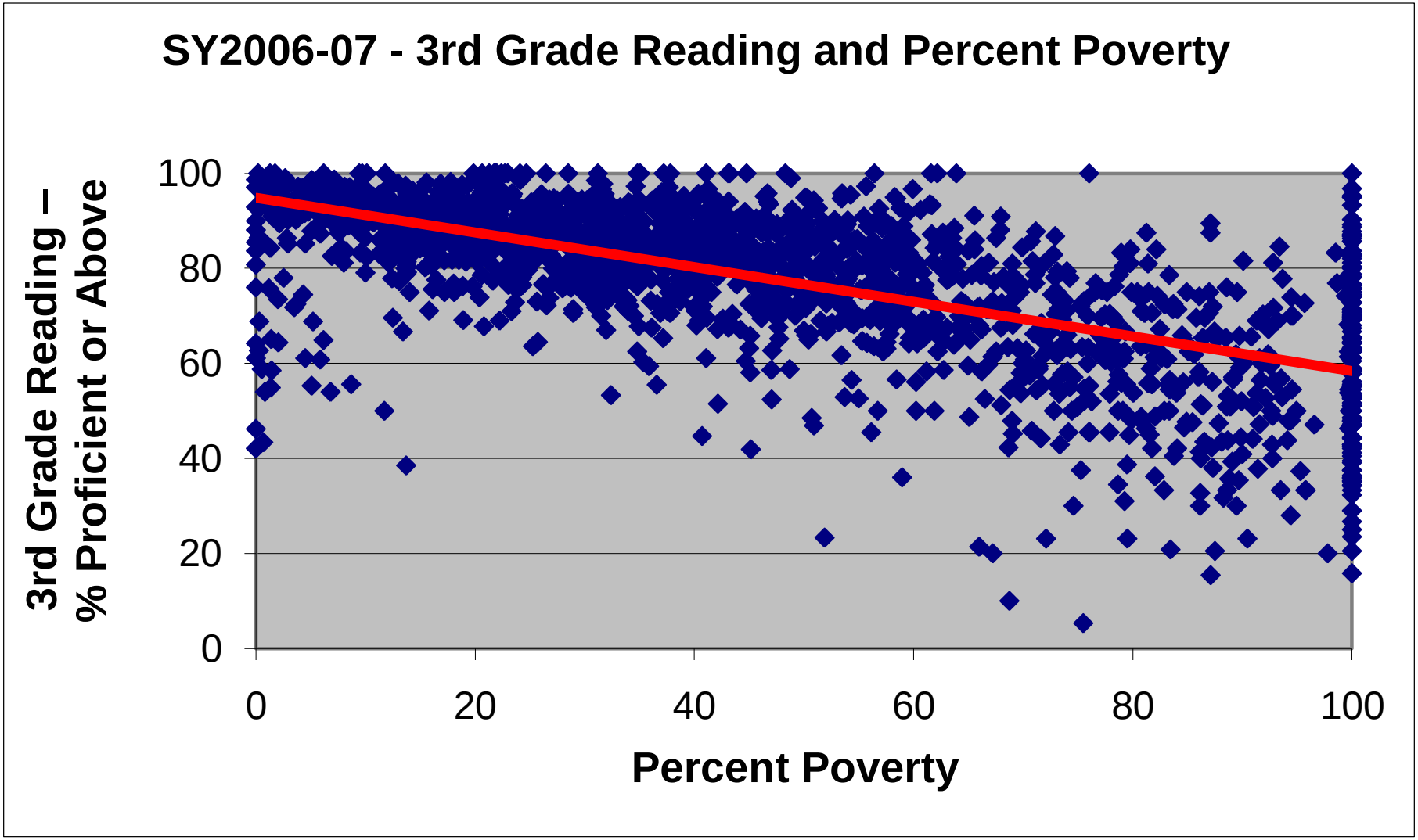
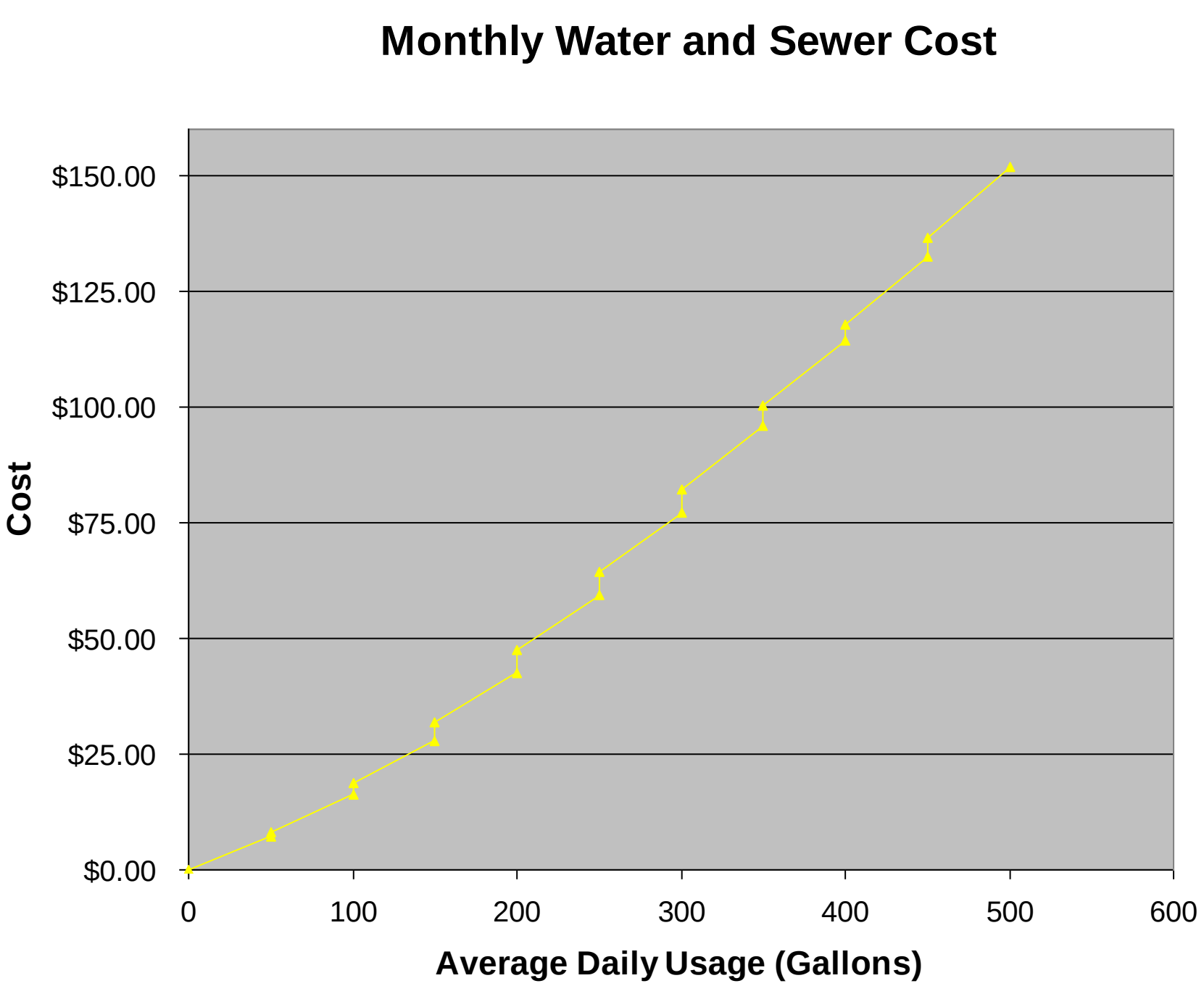
Who Can Interpret This?



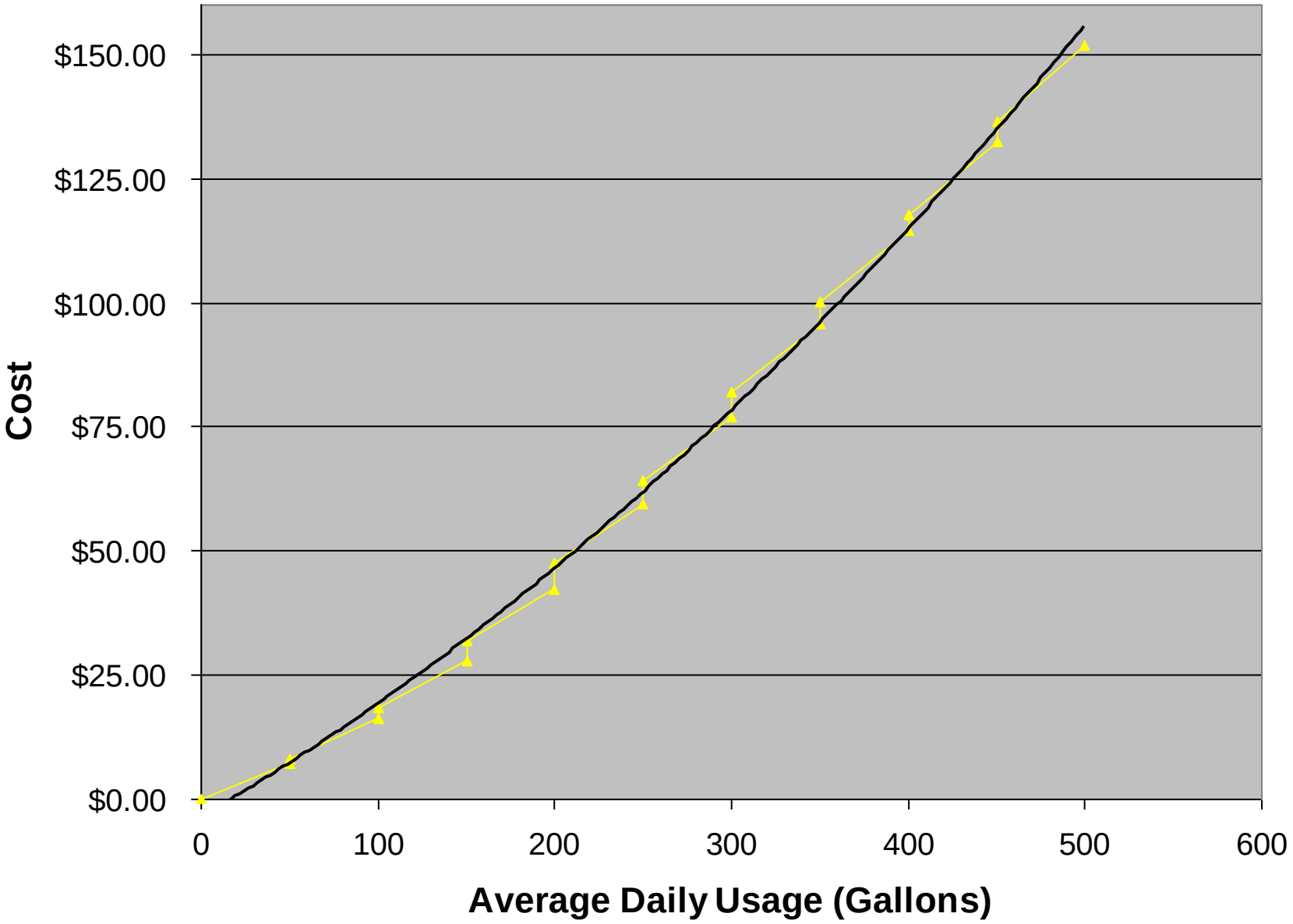
Washington Suburban Sanitary Commission Rate Schedule, July 1, 2008

Average Daily Consumption (Gallons/Day)	Water Rate Per 1,000 Gallons	Sewer Rate Per 1,000 Gallons	Combined Rate Per 1,000 Gallons
0-49	$1.97	$2.77	$4.74
50 - 99	2.21	3.22	5.43
100 - 149	2.42	3.79	6.21
150 - 199	2.71	4.36	7.07
200 - 249	3.17	4.76	7.93
250 - 299	3.43	5.14	8.57
300 - 349	3.63	5.50	9.13
350 - 399	3.79	5.75	9.54
400 - 449	3.94	5.88	9.82
…	…	…	…

Source: http://www.wsscwater.com/service/rates.cfm


Monthly Water and Sewer Cost
$150.00
$125.00
$100.00
$75.00
$50.00
$25.00
$0.00
Cost
0
100
200
300
400
500
600
Average Daily Usage (Gallons)

Monthly Water and Sewer Cost



Rationale for A2E for All

- Algebra 1 and Geometry provide insufficient readiness for college and most careers
- All students need proficiency in A2E for
  - Many careers, with or without college
  - Informed citizenship
  - Individual empowerment
- High school mathematics should open doors
  - But adult decisions often close doors for students
  - After students complete A2E, they have choices
- And again, not your parents' Algebra 2

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What Is Needed?

- Renewed curriculum and instruction
  - Across middle and high school toward a rigorous and relevant A2E
- “Focal Points” for high school
- Support for students are behind
  - To help them catch up

- The CCSS and the Model Pathways are foundational responses to these needs

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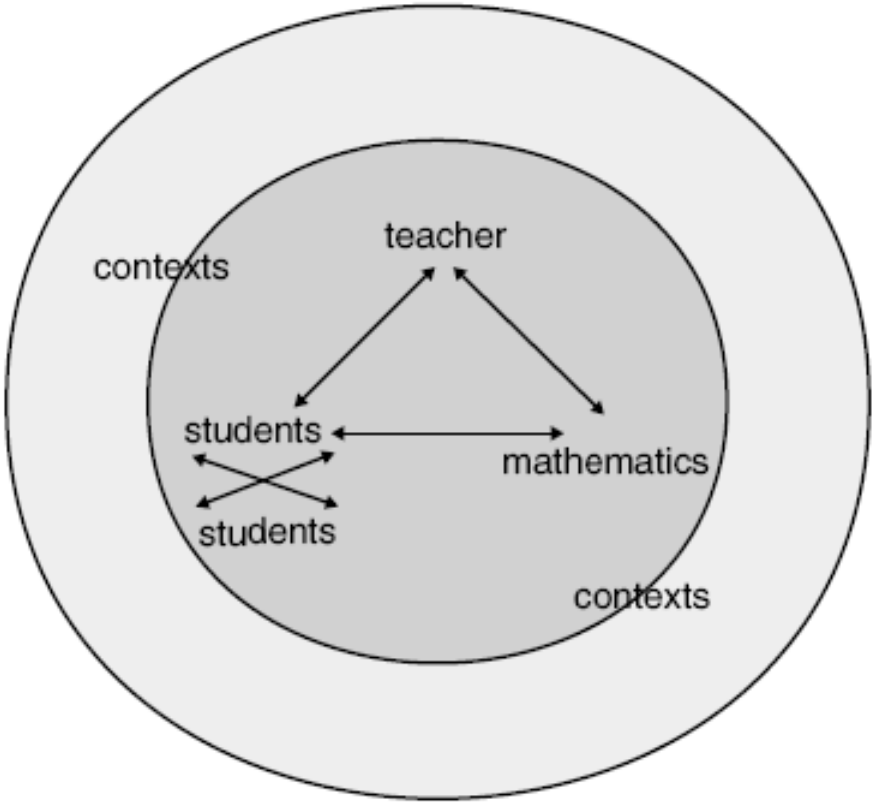
Instruction as Interaction

What matters are the interactions, in classrooms, among the teacher, the students, and the mathematical ideas



Source: Cohen & Ball, 1999, 2000, as cited in NRC, 2001.

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Secondary Mathematics Problems

- Three ways to improve achievement
  - Invest in the knowledge and skill of teacher
  - Change the level of content
  - Change the role of the student in the instructional process.

- Problem of *access*
- Problem of *teaching quality*

- Both of these problems are perpetuated and exacerbated by pervasive myths

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Myth: Basic Skills First

- Myth: Students cannot engage in high-level thinking until they have mastered basic skills
- View is pervasive in high schools, which function primarily as sorting mechanisms
- Students are denied access to quality instruction because of adult judgments
- High schools and their curricula were not designed to teach high-level content to all students

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Myth: Teachers Are Born

- Myth: Teaching ability is a natural predisposition
  - Teaching is an art that cannot be learned
  - The system does not learn; we rarely refine the wisdom of practice
- Teaching is a mass profession
  - Ordinary people doing extraordinary things (Japan)
- Teaching is a skill, with a knowledge base

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High School Math Curriculum

- Algebra 1 and Geometry courses typically
  - Reteach much middle grades content
- Algebra 2 courses typically
  - Reteach Algebra 1
  - Include some statistics and probability
  - Include optional topics
  - Pre-teach Precalculus content
- ***Algebra 2 is two miles wide***
  - *And a quarter inch deep*

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An Algebra Idea Across K-12

- Compare and contrast: patterns, functions, and sequences
- In grades K-8, students study patterns
- In grades 9-11, students study functions
- In grade 12, students might study sequences
- A sequence is a pattern
- A pattern suggests a function
- A sequence is a function with a domain consisting of whole numbers

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A Look Inside the CCSS for High School Mathematics

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CCSS Principles

- Focus
  - Identifies key ideas, understandings and skills for each grade or course
  - Stresses deep learning, which means applying concepts and skills within the same grade or course
- Coherence
  - Articulates a progression of topics across grades and connects to other topics
  - Vertical growth that reflects the nature of the discipline

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CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

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CCSS for High School Mathematics

- Organized in “Conceptual Categories”
  - Number and Quantity
  - Algebra
  - Functions
  - Modeling
  - Geometry
  - Statistics and Probability
- Conceptual categories are not courses
- Additional mathematics for advanced courses indicated by (+)
- Standards with connections to modeling indicated by (⋆)

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Conceptual Category Introduction

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here

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Conceptual Category Overview

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Domain

Making Inferences and Justifying Conclusions

Cluster

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Standards for High School Math

Domain

The Complex Number System N-CN

Cluster

Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Standard

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number

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HS CCSS: Changing Content Emphases

- Number and quantity
  - Number systems, attention to units
- Modeling
  - Threaded throughout the standards
- Geometry
  - Proof for all, based on transformations
- Algebra and functions
  - Organized by mathematical practices
- Statistics and probability
  - Inference for all, based on simulation

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Calls from Business

- Students need to solve interdisciplinary problems
  - So teachers need to work across disciplines
  - And across algebra, geometry, and data analysis
- Students need to learn to collaborate
  - So teachers need to collaborate
  - And take responsibility for **all** students

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A Look Inside the Model Pathways

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High School Mathematical Pathways

Typical in U.S.

- Two main pathways:
  - Traditional: Two algebra courses and a geometry course, with statistics and probability in each
  - Integrated: Three courses, each of which includes algebra, geometry, statistics, and probability

Typical outside U.S.

- Both pathways:
  - Complete the Core in the third year
  - Include the same “critical areas”
  - Require rethinking high school mathematics
  - Prepare students for a menu of fourth-year courses

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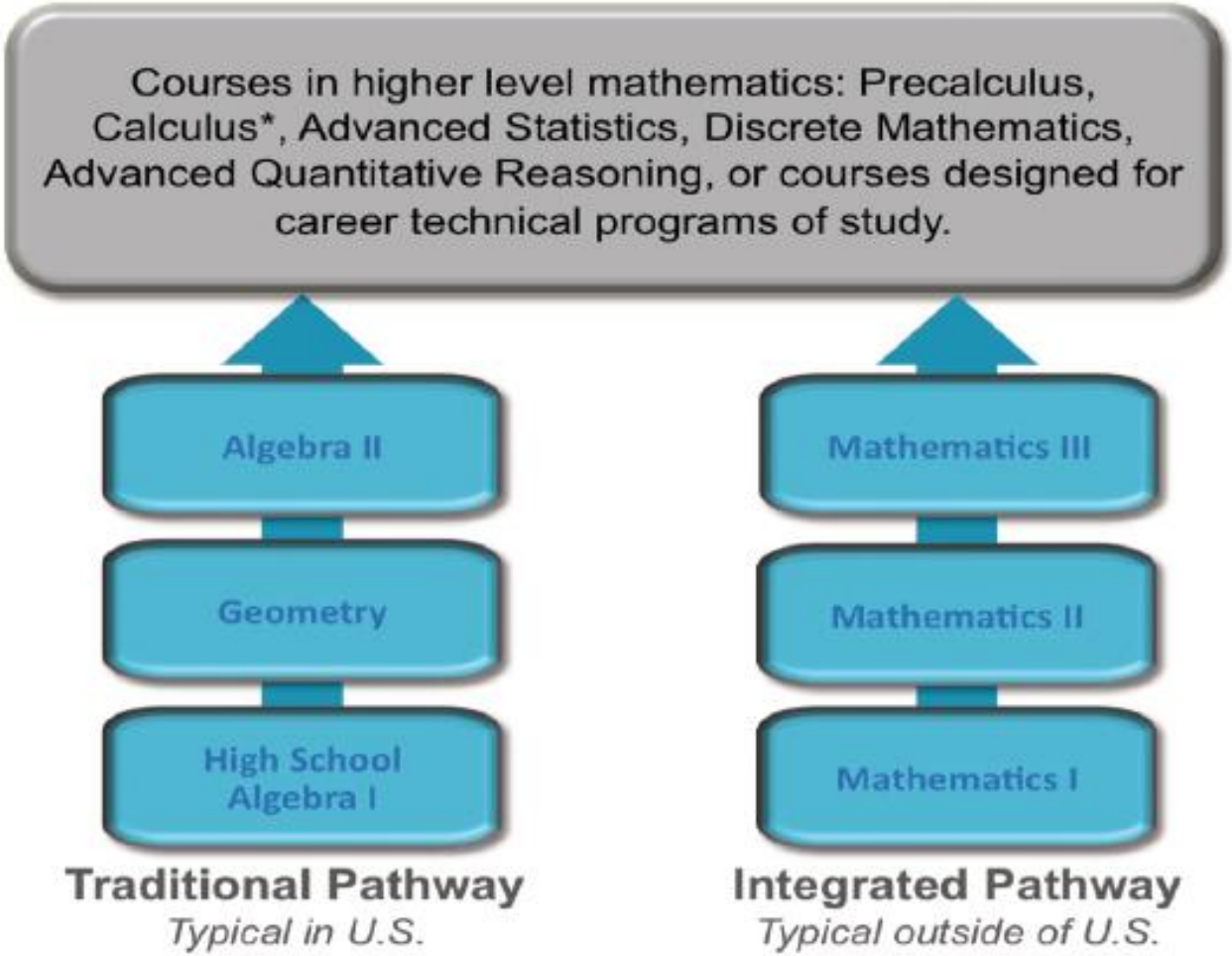
Two Main Pathways



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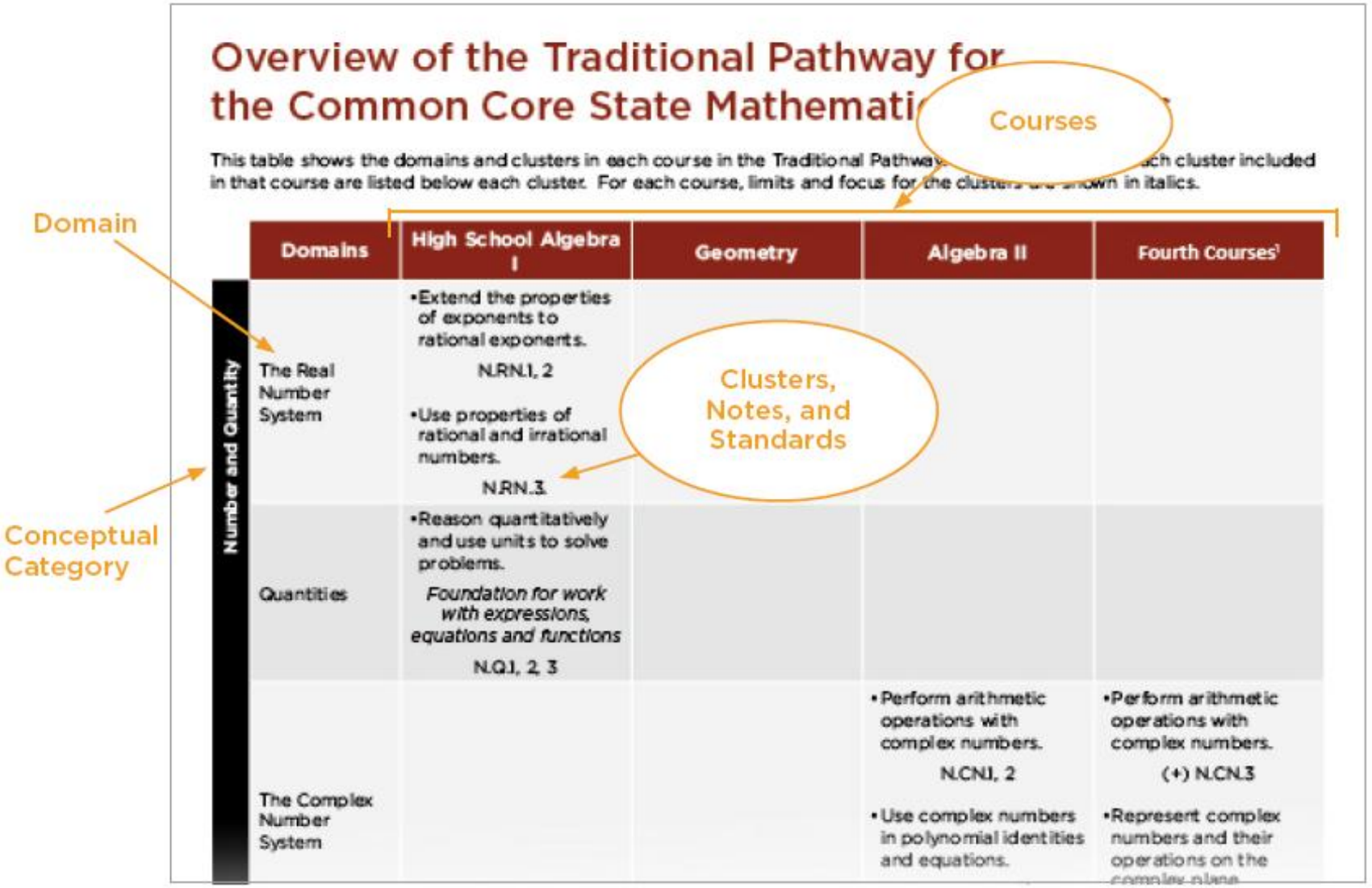
Pathway Overview



Course Overview: Critical Areas (units)

Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics 1 uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe

Course Detail by Unit (critical area)

Algebra 1 in Eighth Grade?

- This is the wrong question
- The Grade 8 CCSS includes much of Algebra 1 *for all students*
- Model Pathway H.S. Algebra 1 builds on it
  - So *do not skip* the Grade 8 CCSS
- Two “compacted” Pathways for grades 7-9 provide paths to Calculus in high school
- Offer “compacted” courses to students who are willing to do the extra work
  - And make sure students succeed

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All Students Means ALL Students

- How well are you serving the following groups?
  - High-achieving students
  - Middle-achieving students
  - Low-achieving students
- District goals sometimes consider only the state assessments
- Do you spend time considering progress of and projections for individual students?

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High-Achieving Students

- What percentage of your students take AP courses?
- How successful are your calculus offerings?
  - High school calculus should be AP Calculus.
- What happens to accelerated students?
  - Do they take mathematics every year?
  - If not, why not?
  - Are they successful?
- What about radically accelerated students?

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Middle-Achieving Students

- How many of your seniors are taking significant (non-remedial) mathematics?
- Do you have fourth-year alternatives to Precalculus?
  - AP Statistics
  - Advanced Quantitative Reasoning
  - National work:

    http://math.arizona.edu/~ime/2008-09/1018_retreat.html

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Low-Achieving Students

- How many of your seniors are taking low-level mathematics?
- Does your program help low-achieving students get back on track?
  - You can't help students catch up by slowing them down
- A guiding principle for intervention:
  - Give all students access to the regular curriculum, and provide differentiated instruction and support
- How many tracks do you need?

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Prealgebra at High School?

- Prealgebra should not count as high school mathematics
  - Preparation for current HS graduation tests
  - College admissions requirements (and NCAA)
  - Reaching college and career readiness
- When students are behind
  - Give them access to the regular curriculum *and* extra support
  - Response to Intervention

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Closing Thought

- “These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.”
  - (CCSS, 2010, p. 5)

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