Number Talks

for

Improving Mathematics Learning

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Underlying Principle

- "*Everyone is good at mathematics because everyone can think. And mathematics is about thinking.*"
  - Yeap Ban Har, National Institute of Education, Singapore.
- Corollary 1: Strategies that attempt to remove thinking from learning are bound to fail in the long run.
- Corollary 2: When learning is effective, "getting the right answer" is but a small piece of the work.

CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Let's Do Some Mathematics

What Errors Might Students Make?

$$\begin{array}{r} 73 \\ -\,59 \\ \hline \end{array}$$

$$\frac{1}{2}+\frac{2}{3}$$

$$\frac{3\cdot 11}{2\cdot 3\cdot 7\cdot 11}$$

$$2\tfrac{1}{3}\times 3\tfrac{1}{4}$$

$$(3\sqrt{2})(5\sqrt{2})$$

$$(x+y)^2$$

$$\frac{a+2}{a}$$

$$\frac{\sin \pi/2}{\cos \pi}$$

Possible Errors

$$\begin{array}{r} 73 \\ -\,59 \\ \hline 26 \end{array}$$

$$\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$$

$$\frac{\cancel{3}\cdot\cancel{11}}{2\cdot\cancel{3}\cdot 7\cdot\cancel{11}}=0$$

$$2\tfrac{1}{3}\times 3\tfrac{1}{4}=6\tfrac{1}{12}$$

$$(3\sqrt{2})(5\sqrt{2}) = 3\cdot 5+3\sqrt{2}+5\sqrt{2}+2$$

$$(x+y)^2=x^2+y^2$$

$$\frac{\cancel{a}+2}{\cancel{a}}=2$$

$$\frac{\sin\cancel{\pi}/2}{\cos\cancel{\pi}}$$

Why Do Students Make Errors?

- Many errors are inappropriate uses of correct procedures
  - Shortcuts (e.g., cancel, FOIL, cross multiply) are especially easy to misapply
  - See Phil Daro's video, Against Answer Getting
- Many errors are due to insufficient attention to the meanings of symbols
  - e.g., place value, fraction, mixed number, exponent
- Many errors indicate insufficient "number sense"
- Errors are usually "sensible" not "arbitrary"

- Number talks are a way of restoring meaning and building number sense

Let's Try Some Number Talks

$73 - 59$ 17×25 $2\frac{1}{3} \times 3\frac{1}{4}$

Note: Try to do these computations mentally.

When you describe your strategies to the whole group, we will "record your thinking" in order to make that thinking visible.

Benefits of Number Talks

- Some number-talk strategies reflect general principles
  - e.g., about place value or properties of operations
- Some number-talk strategies work well for the particular numbers at hand
  - *Ad hoc* strategies do not replace general algorithms but they can promote algebraic reasoning
- High achieving students do these things "naturally"
  - Make these strategies explicit: teach them
  - Better yet, let students teach each other

Key Ideas

- What is the most underutilized resource in K-12 schools today?
  - The students' heads
- The most important thing you can do is convince your students that their thinking matters
- Number talks can serve that goal

Day Two Opening

Guiding Principles for Number Talks

1. All students have mathematical ideas worth listening to.
2. Through our questions, we seek to understand students' thinking.
3. We encourage students to explain their thinking conceptually rather than procedurally.
4. Mistakes provide opportunities to look at ideas that might not otherwise be considered.
5. The efficiency of a strategy depends upon the thinking of each individual.
6. We seek to create a learning environment where all students feel safe.
7. One of our most important goals is to help students develop social and mathematical agency.
8. Mathematical understandings develop over time.
9. Confusion and struggle are natural, necessary, and even desirable.
10. We value and encourage a diversity of ideas.

(Adapted from Humphreys & Parker, 2015, pp. 25-31)

On Problem Solving

- Stacy, a high school Algebra 2 teacher, asked:
  - "How do you feel when you try to solve a problem you haven't seen before?"
- Her students' responses:
  - Jamie: Very uncertain because I think I could be using the wrong kind of equation or way of solving it.
  - Alex: When I see a problem that I don't know, I don't attempt it at all because I don't want to be wrong.
  - Alicia: I feel like giving up when I don't already know how to do it because I rely on my teacher to explain it.

(Humphreys & Parker, 2015, pp. 30)

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Discussion Questions

- The purpose of a number-talk problem is not the answer.
  - What is the purpose?
  - How do we help students value this?

- Asking every student to learning every strategy is overwhelming (for both students and teachers).
  - What is a more reasonable goal for learning?

On Seeing Mathematically

- “I’ve come to believe that my job is not to teach my students to see what I see. My job is to teach them to see.”
  - Ruth Parker

(Humphreys & Parker, 2015, pp. 31)

Day Two Closing

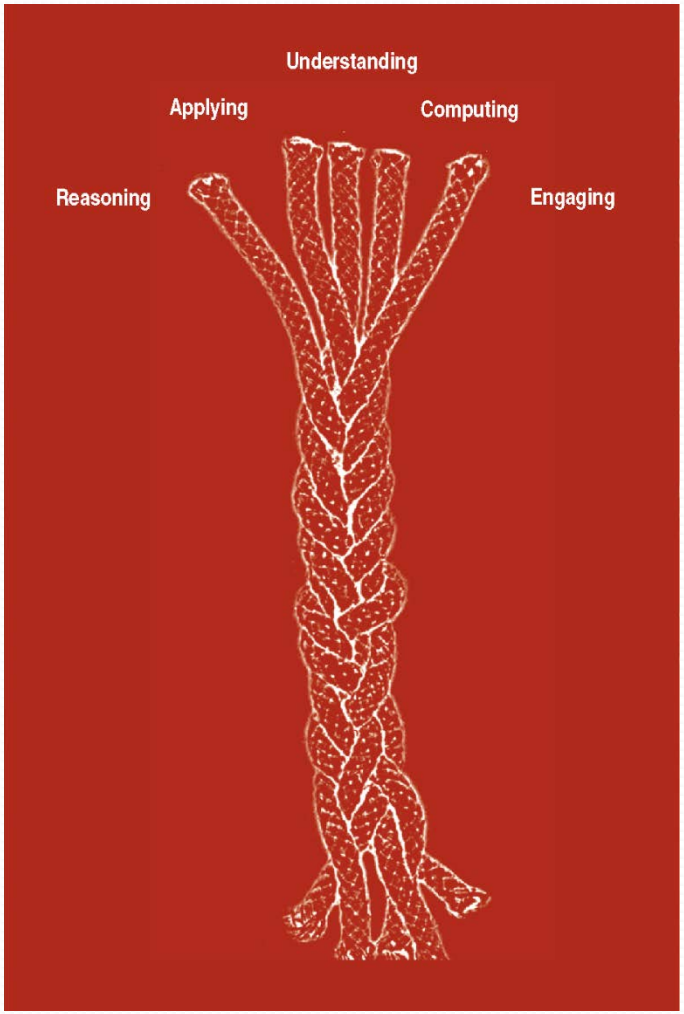
Comments Up Front

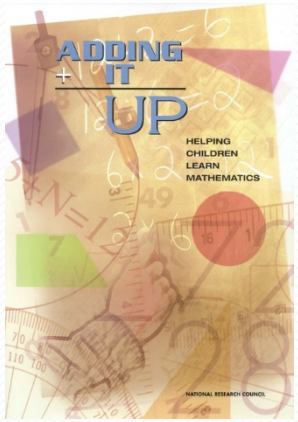
- Kudos to all of you for taking the lead on this important work
  - The engagement is phenomenal—as is the willingness to struggle
  - The learning and sharing is evident
  - The collaborative atmosphere is contagious
- Kudos to your facilitators for stepping up to help guide the work
  - They provide considerable expertise, commitment, and leadership
  - And they are a great team!
- Kudos to Idaho State Board of Education, Cory, and Julie for providing these opportunities
  - This is the right work: Supporting teachers working together to improve teaching and learning

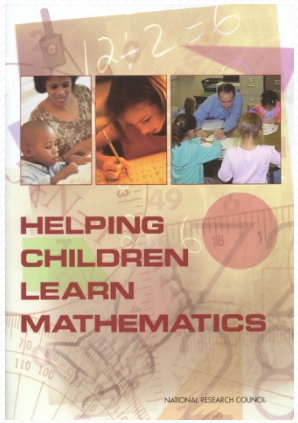
Benefits of Number Talks

- Number talks involve breaking things apart and putting them back together
  - A very powerful idea that undergirds much mathematics
- Number talks provide insight into student thinking
  - To inform your teaching
- Number talks can be introduced as a new routine
- Number talks can be incorporated into existing routines
- Number talks promote
  - Flexible thinking and problem solving
  - Fluency with number and operation
  - Reasoning and sense-making
  - *An authentic view of mathematics*

Mathematical Proficiency







Strands of Mathematical Proficiency

1. **Understanding:** Comprehending mathematical concepts, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean.
2. **Computing:** Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.
3. **Applying:** Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.
4. **Reasoning:** Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.
5. **Engaging:** Seeing mathematics as sensible, useful, and doable—*if* you work at it—and being willing to do the work.

Thoughts for Successful Number Talks

1. Being comfortable with plenty of wait time.
2. Practicing “graduated pressing”
3. Thinking together
4. Learning to listen
5. Doing Number Talks regularly
6. Encouraging clarity of academic language during students’ sharing
7. Recording thinking
8. Trying to get students to talk with one another without going through you
9. Making the most of multiple answers
10. Helping students learn to express themselves more clearly
11. Having a backup plan in place
12. Knowing when it’s okay to share your way of thinking during Number Talks
13. “Nudging” students to move beyond traditional algorithms

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The Division Problem

- The question was, “What is 42 ÷ 7?”
- Here is a second grader’s solution. He did not write anything. This is what he said:

I don’t know how many 7s are in 42, but I do know:

40 ÷ 10 = 4

4 × 3 = 12

12 + 2 = 14

14 ÷ 7 = 2

2 + 4 = 6

The answer is 6.

40 = 4×10

= 4×(7 + 3)

= 4×7 + 4×3

So 4 7s is less than 40 by 4×3, or 12

So 4 7s is less than 42 by 14

… which is 2 more 7s

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Closing Thoughts

- I hope you all see the promise of Number Talks for
  - Making student thinking visible
  - Improving attention to meaning
  - Improving number sense
  - Improving reasoning
- You can do this!
- You are already doing some of this
- We can all improve our practice

- *So support each other—work together!*

- Slides will be available at http://bradfindell.com