Implementing the Common Core State Standards for Mathematics

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What's New with the CCSS?

- Internationally benchmarked standards
- Common across 40+ states
- College and career readiness for all
- Focus and coherence
- And all students means ALL students

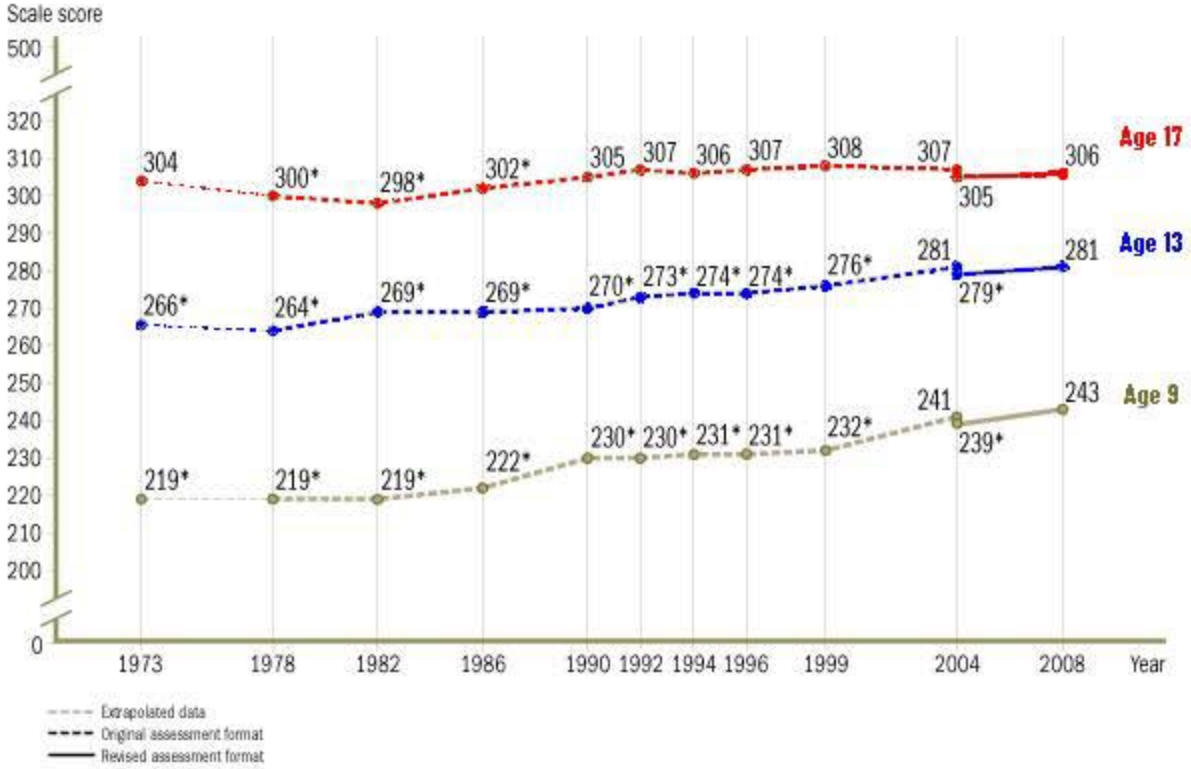
Underlying Principle

- "*Everyone is good at mathematics because everyone can think. And mathematics is about thinking.*"
  - Yeap Ban Har, National Institute of Education, Singapore.
- Corollary 1: Strategies that attempt to remove thinking from learning are bound to fail in the long run.
- Corollary 2: When learning is effective, "getting the right answer" is but a small piece of the work.

Overview

- A look inside the system
- Toward college and career readiness
- A look inside the CCSS for Mathematics
- About serving all students
- Implementation suggestions and resources

Trends in National Average Mathematics Scores



Since 1973	Since 2004
↔	↔
↑ 15 points	↑ 3 points
↑ 24 points	↑ 4 points

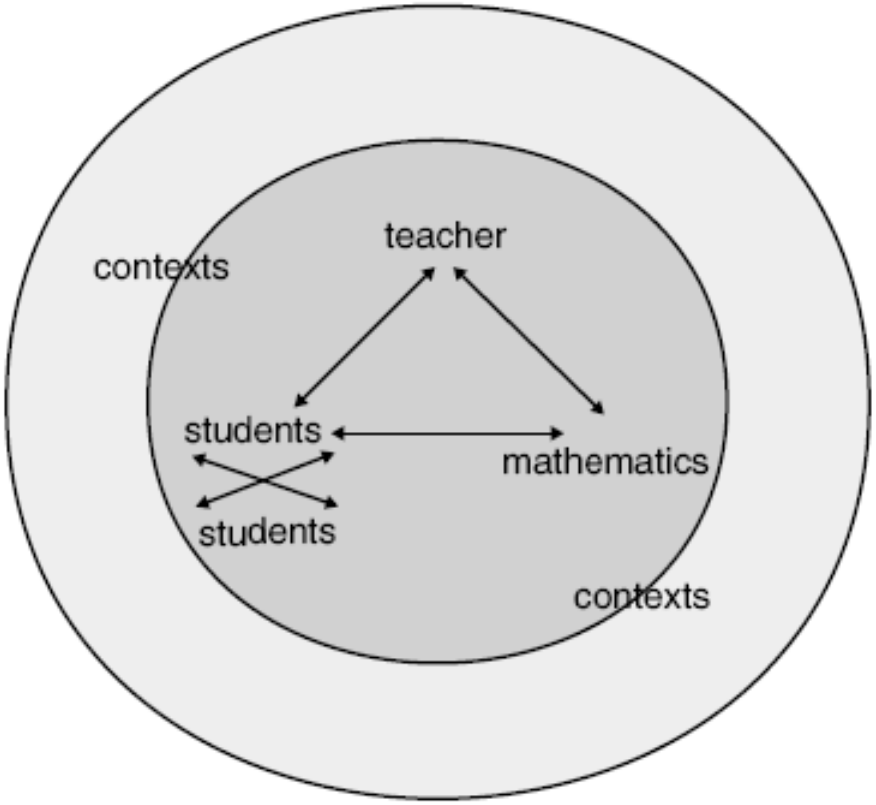
↑ Indicates the score was higher in 2008.
↔ Indicates there was no significant change in the score in 2008.

Mathematics Achievement Trends

- Achievement is up by many indicators
  - Significant growth in grades 4 and 8
  - High school diploma, math course taking
  - College attendance, college completion
- High school achievement is flat
  - U.S. 15-year-olds lag in applying math
  - Poor results on H.S. end-of-course exams
  - College remediation rates remain high
- **Today's world demands more**

Instruction as Interaction

What matters are the interactions, in classrooms, among the teacher, the students, and the mathematical ideas



Source: Cohen & Ball, 1999, 2000, as cited in NRC, 2001.

(Secondary) Mathematics Problems

- Three ways to improve achievement
  - Invest in the knowledge and skill of teacher
  - Change the level of content
  - Change the role of the student in the instructional process.

- Problem of *access*
- Problem of *teaching quality*

- Both of these problems are perpetuated and exacerbated by pervasive myths

Myth: Basic Skills First

- Myth: Students cannot engage in high-level thinking until they have mastered basic skills
- View is pervasive in high schools, which function primarily as sorting mechanisms
- Students are denied access to quality instruction because of adult judgments
- High schools and their curricula were not designed to teach high-level content to all students

Myth: Natural Teachers Are Born

- Myth: Teaching ability is a natural predisposition
  - Teaching is an art that cannot be learned
  - The system does not learn; we rarely refine the wisdom of practice
- Teaching is a mass profession
  - Ordinary people doing extraordinary things (Japan)
- Teaching is a skill, with a knowledge base

College and Career Readiness

- More states are requiring Algebra 2 or its equivalent (A2E)
  - A proxy for college and career readiness
- CCSS definition of college and career readiness:
  - All standards not indicated by (+)
- We need to make A2E rigorous, relevant, and attainable
  - Your parents' Algebra 2 will not do

- *But many teachers do not support this goal*

A Look Inside the CCSS for Mathematics

CCSS Principles

- Focus
  - Identifies key ideas, understandings and skills for each grade or course
  - Stresses deep learning, which means applying concepts and skills within the same grade or course
- Coherence
  - Articulates a progression of topics across grades and connects to other topics
  - Vertical growth that reflects the nature of the discipline

CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Grade Level Overview

Cross-cutting themes

Mathematics | Grade 2

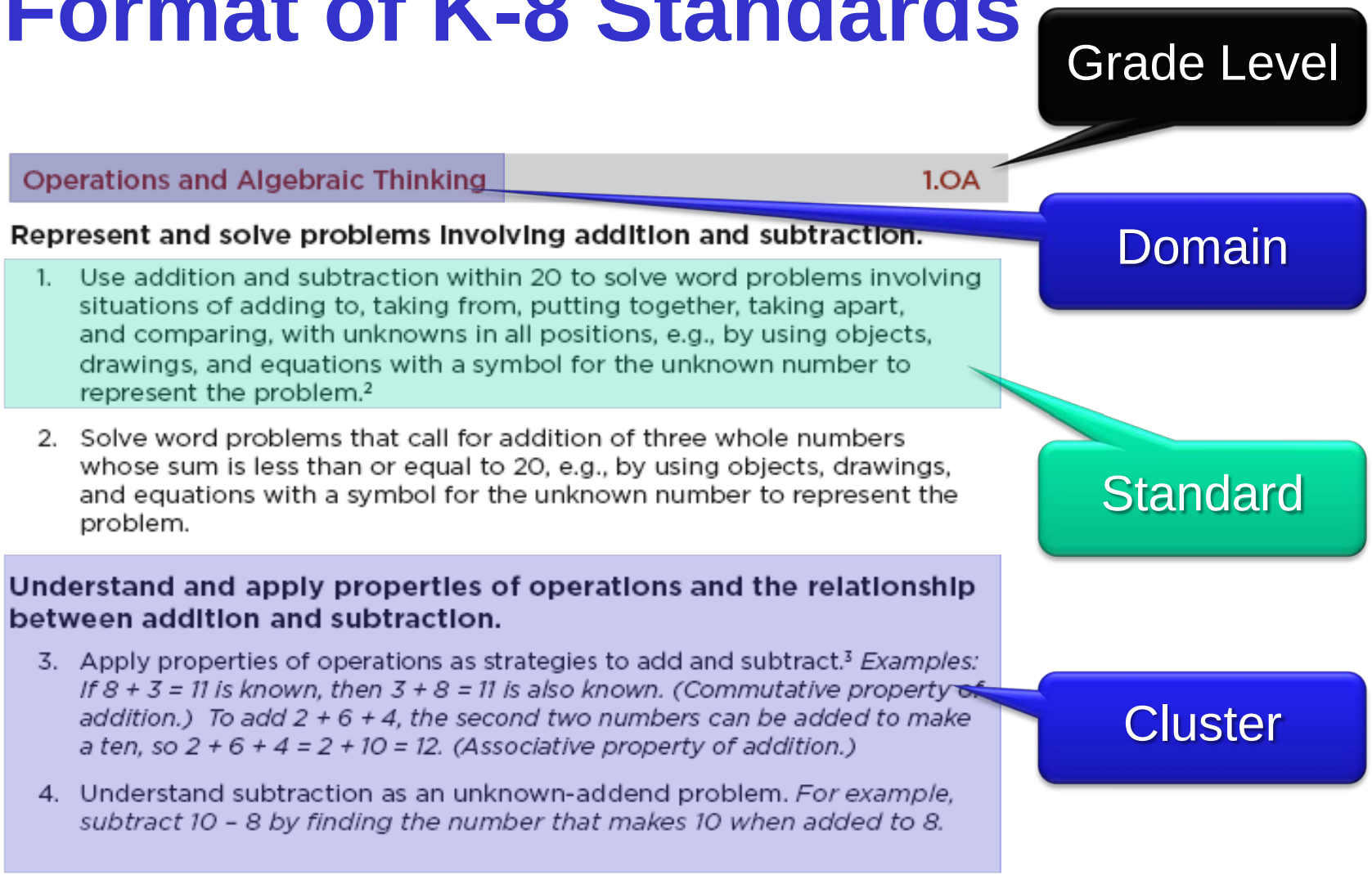
In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Critical Area of Focus

(1) Students extend their understanding of the base-ten system. T
includes ideas of counting in fives, tens, and multiples of hundreds, t
and ones, as well as number relationships involving these units, in
comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000

Format of K-8 Standards



K-8 CCSS Changing Content Emphases

- Primary focus on number in grades K-5
- Fractions as numbers on the number line, beginning with unit fractions
- Fluency with standard algorithms, supported by strategies based in place value
- Much statistics in grade 6-8
- Much algebra and geometry in grades 7-8

CCSS for High School Mathematics

- Organized in “Conceptual Categories”
  - Number and Quantity
  - Algebra
  - Functions
  - Modeling
  - Geometry
  - Statistics and Probability
- Conceptual categories are not courses
- Additional mathematics for advanced courses indicated by (+)
- Standards with connections to modeling indicated by (★)

Conceptual Category Introduction

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here

Conceptual Category Overview

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Domain

Making Inferences and Justifying Conclusions

Cluster

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for High School Math

Domain

Cluster

Standard

Advanced

The Complex Number System **N-CN**

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number

High School Mathematics Today

- Algebra 1 and Geometry courses typically
  - Reteach much middle grades content
- Algebra 2 courses typically
  - Reteach Algebra 1
  - Include some statistics and probability
  - Include optional topics
  - Pre-teach Precalculus content
- ***Algebra 2 is two miles wide***
  - *And a quarter inch deep*

HS CCSS Changing Content Emphases

- Number and quantity
  - Number systems, attention to units
- Modeling
  - Threaded throughout the standards
- Geometry
  - Proof for all, based on transformations
- Algebra and functions
  - Organized by mathematical practices
- Statistics and probability
  - Inference for all, based on simulation

CCSS Domain Progression

<table>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting & Cardinality</td><td colspan="9"></td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td><td rowspan="2">Number & Quantity</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations – Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="9">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics & Probability</td></tr>
</table>

Designing Mathematics Programs for All Students

All Students Means ALL Students

- How well are you serving the following groups?
  - High-achieving students
  - Middle-achieving students
  - Low-achieving students
- District goals sometimes consider only the state assessments
- Do you spend time considering progress of and projections for individual students?

High-Achieving Students

- What percentage of your students take AP and IB courses?
- How successful are your calculus offerings?
  - High school calculus should be AP Calculus.
- What happens to accelerated students?
  - Do they take mathematics every year?
  - If not, why not?
  - Are they successful?
- What about radically accelerated students?

Middle-Achieving Students

- How many of your seniors are taking significant (non-remedial) mathematics?
- Do you have fourth-year alternatives to Precalculus?
  - AP Statistics
  - Advanced Quantitative Reasoning
  - National work:

    http://math.arizona.edu/~ime/2008-09/1018_retreat.html

Low-Achieving Students

- How many of your seniors are taking low-level mathematics?
- Does your program help low-achieving students get back on track?
  - You can't help students catch up by slowing them down
- A guiding principle for intervention:
  - Give all students access to the regular curriculum, and provide differentiated instruction and support
- How many tracks do you need?

CCSSM and Acceleration

- The CCSS for Mathematics represent significant curricular acceleration in grades K-8
  - Much of Algebra 1 and Geometry are in the middle grades
  - Many “accelerated” programs will no longer be ahead
  - The CCSS for Grade 8 is a reasonable, internationally benchmarked response to Algebra for all in grade 8
- Accelerating large percentages of students much beyond the CCSS for K-8 is probably unwise
- The CCSSM for high school include much advanced content and much new content for all students
- *So we need to rethink mathematics, grades 6-12*

Algebra 1 in Eighth Grade?

- This is the wrong question
- The Grade 8 CCSS includes much of Algebra 1 *for all students*
- Model Pathway H.S. Algebra 1 builds on it
  - So *do not skip* the Grade 8 CCSS
- Two “compacted” Pathways for grades 7-9 provide paths to Calculus in high school
- Offer “compacted” courses to students who are willing to do the extra work
  - And make sure students succeed

Prealgebra at High School?

- Prealgebra should not count as high school mathematics
  - Preparation for current HS graduation tests
  - College admissions requirements (and NCAA)
  - Reaching college and career readiness
- When students are behind
  - Give them access to the regular curriculum *and* extra support
  - Response to Intervention

Implementation Suggestions, Challenges, and Resources

Research-Based Principles

- Implementation matters
  - Variation within a model is greater than the variation between models
  - Adoption of standards, programs, or textbooks merely opens the door
- High-quality professional development
  - Focuses on the content the teachers are teaching
  - Draws on curricular materials teachers are using
  - Involves analyzing student work
  - Takes time

Maintain Focus and Coherence

- Implementation may miss the point
  - Readers might not see focus and coherence
  - Strategies may be counterproductive
- The goal is coherence in curriculum, instruction, and learning
  - Standards are taken as atoms, but the power is in the bonds (Jason Zimba)
  - Think in chapters, not lessons (Phil Daro)

Challenges and Responses

- Crosswalk documents may encourage rearrangements of low-quality curricular materials and frameworks
  - Aim for focused, forward-thinking crosswalk documents at the level of clusters or big ideas
- Unpacking standards (*a la* backwards design) may perpetuate the atomized check-list mentality
  - Unpack clusters of standards via descriptive paragraphs
- Response to Intervention may be misused to sort students into groups that receive fundamentally different instruction
  - Begin with high-quality, Tier 1 instruction for all students
- Data-driven decision making may remain only about numbers
  - Use data to provoke targeted discussions about instruction

Challenges and Responses

- Formative assessment may be misconstrued as a task bank
  - Formative assessment must provide insight into student thinking
- Professional development may be largely generic and unfocused
  - Develop strategies for content-based professional learning communities
- Publishers may add chapters to existing materials
  - Insist on materials with focus and coherence
- Local control and limited resources may create excuses
  - Share and borrow materials
  - Leverage resources
  - Take advantage of the assessment consortia

What Should Districts Do Now?

- Get to know the CCSSM through Professional Learning Communities
  - Use the “critical areas of focus”
  - Take a “progressions view”
- Begin developing the Mathematical Practices
- Develop support structures for struggling students
  - *a la* Response to Intervention
- Identify transitional changes for 2012-13
- Be skeptical of easy alignment and quick fixes
- Watch for new opportunities and resources

Ohio's Model Curriculum for the CCSS

Mathematics Model Curriculum

Ohio | Department of Education

This is the March 2011 version of the Grade 3 Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, misconceptions, and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multi-state initiatives to develop common content elaborations and learning expectations.

Grade 3	
Domain	**Cluster**
Operations and Algebraic Thinking	• Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship between multiplication and division. • Multiply and divide within 100. • Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Number and Operations in Base Ten	• Use place value understanding and properties of operations to perform multi-digit arithmetic.

Ohio's Model Curriculum for the CCSS

Instructional Strategies and Resources

Instructional Strategies

Students need to understand the part/whole relationships in order to understand the connection between multiplication and division. They need to develop efficient strategies that lead to the big ideas of multiplication and division. These big ideas include understanding the properties of operations, such as the commutative and associative properties of multiplication and the distributive property. The naming of the property is not necessary at this stage of learning; it's the idea that is important.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This is called unitizing which requires students to count groups, not just objects. They see the whole as a number of groups of a number of objects. This strategy is a foundation for multiplication in that students should make a connection between repeated addition and multiplication.

As students create arrays for multiplication using objects or drawing on graph paper, they may discover that three groups of four and four groups of three yield the same results. They should observe that the arrays stay the same, although how they are viewed changes. Provide numerous situations for students to develop this understanding.

To develop an understanding of the distributive property, students need decompose the whole into groups. Arrays can be used to develop this understanding. To find the product of 3 × 9, students can decompose 9 into the sum of 4 and 5 and find 3 × (4 + 5).

Ohio's Model Curriculum for the CCSS

Instructional Resources/Tools

Region or area models
Length or measurement models
Grid or dot paper (draw pictures to explore fraction ideas)
Set models
Geoboards
Fraction bars or strips

National Library of Virtual Manipulatives
- Fractions – Naming – Write the fraction corresponding to the highlighted portion of a shape.
- Fractions – Visualizing – Illustrate a fraction by dividing a shape and highlighting the appropriate parts.
- Fractions – Parts of a Whole – Relates parts of a whole unit to written description and fraction.

From the National Council of Teachers of Mathematics, Illuminations: Fun with Fractions: Investigating Equivalent Fractions with Relationship Rods - Students investigate the length model by working with relationship rods to find equivalent fractions. Students develop skills in reasoning and problem solving as they explain how two fractions are equivalent (the same length).

Transparencies you can make to show students how equivalent fractions measure up on the number line.
http://mathforum.org/paths/fractions/seeing.equiv.html

From the National Council of Teachers of Mathematics, Illuminations: Fun with Fractions - In this unit, students explore relationships among fractions through work with the length model. This early work with fraction relationships helps students make sense of basic fraction concepts and facilitates work with comparing and ordering fractions and working with equivalency.

Learn Fractions with Cuisenaire Rods
- template to create Cuisenaire rods.
- equivalent fractions

Ohio's Model Curriculum for the CCSS

Common Misconceptions

The use of terms like "round up" and "round down" confuse many students. For example, the number 37 would round to 40 or rounds up. The digit in the tens place is changed from 3 to 4 (rounds up). This thinking with student is what causes the problem when applied to rounding down. The number 32 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use either a number line to visualize the placement of the number and/or ask the question: What tens are 32 between and which one is it closer to? Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding.

Resources for Implementation

- Standards Progressions
  - Describes how ideas connect and grow across grades
  - Technical appendix (Zimba) highlights structural features that are not highly visible in the document
  - See http://commoncoretools.wordpress.com
- Illustrative Mathematics Project
  - Review board and task vetting process
  - See http://www.illustrativemathematics.org/
- Curriculum analysis toolkit
  - Partnership between CCSSO and NCSM, led by Bill Bush

Organizations Developing Resources

- Smarter Balanced Assessment Consortium
- Partnership for Assessment of Readiness for College and Careers
- National Council of Teachers of Mathematics
- National Council of Supervisors of Mathematics
- Association of Mathematics Teacher Educators
- Association of State Supervisors of Mathematics
- Council of Chief State School Officers
- National Governors Association

- *These organizations are collaborating as the Mathematics Common Core Coalition*

Implementation Questions for You

- Can we empower mathematics teachers to make necessary changes?
  - Curriculum, instruction, support, programs, …
- Can we get the incentives right?
  - So that teachers will regularly work together to reach more students more of the time
  - So that we all learn from and with our best teachers
- Can we bring mathematics leadership to the decision-making table?
  - So that school-improvement efforts focus on long-term improvements not short-term fixes

Closing Thought

- “These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.”
  - (CCSS, 2010, p. 5)