Implementing

the

Common Core State Standards

for

Secondary Mathematics, 7-12

Boise, ID

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Bradford R. Findell, PhD
brad@findell.org
http://www.bradfindell.com

Underlying Principle

- "*Everyone is good at mathematics because everyone can think. And mathematics is about thinking.*"
  - Yeap Ban Har, National Institute of Education, Singapore.
- Corollary 1: Strategies that attempt to remove thinking from learning are bound to fail in the long run.
- Corollary 2: When learning is effective, "getting the right answer" is but a small piece of the work.

Overview

- Key messages from Response to Intervention (RtI)
- Key messages from the CCSS for Mathematics
- A look inside the CCSS for Mathematics
- Assessment and program considerations
- Implementation resources and suggestions
- Questions

Major Themes

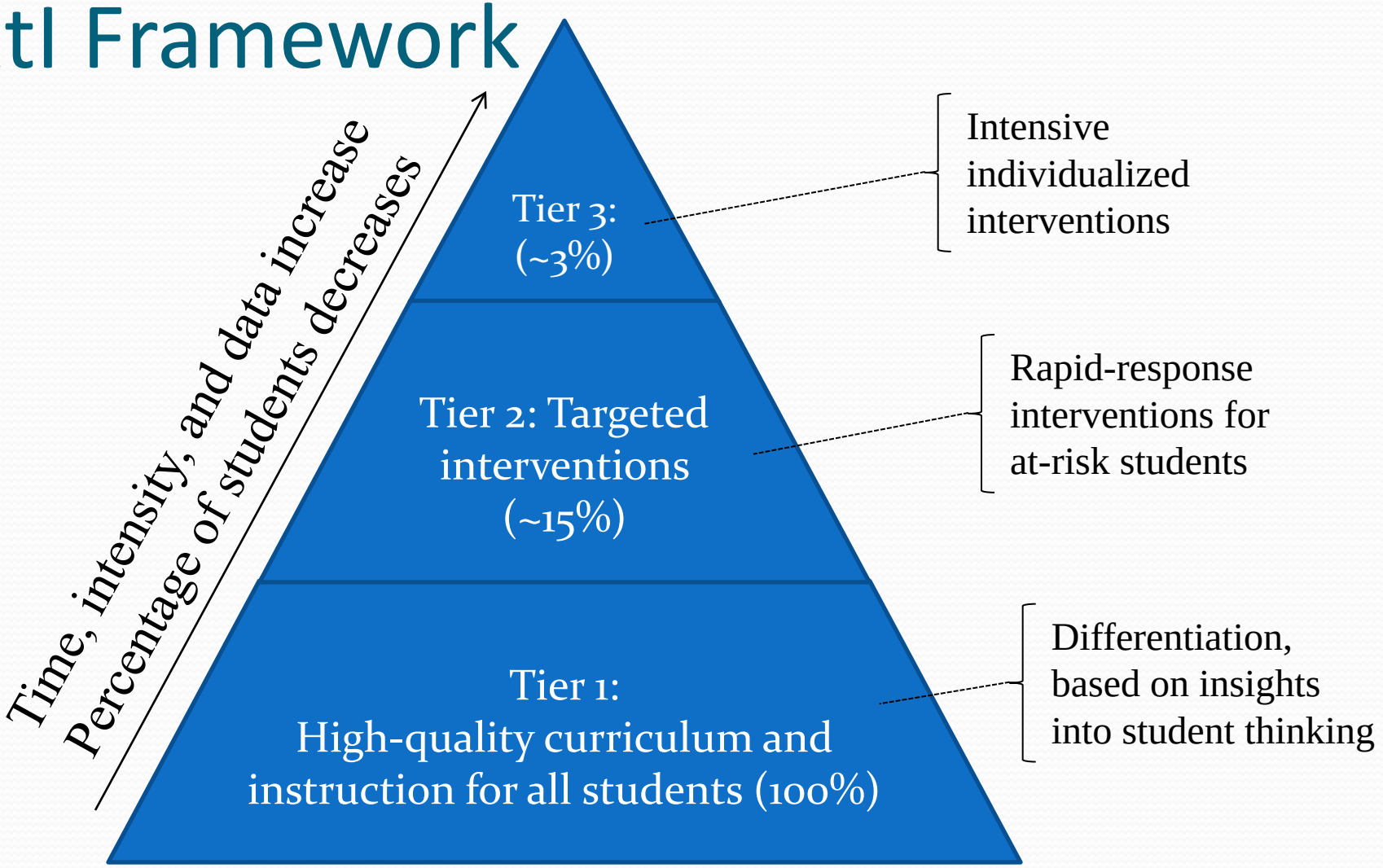
- All students means ALL students
- The work is about improving instruction, which requires that teachers (all teachers) collaborate to reach more students more of the time
- Common messages among current initiatives
  - Common Core State Standards
  - Formative Assessment
  - Response to Intervention
  - School Turnaround
  - …

Key Messages from Response to Intervention

What Is RtI?

- RtI is about establishing a school-wide system for allocating instructional resources where they are needed
  - Give all students access to the regular curriculum AND provide differentiated instruction and support
  - Some students are 15 minutes behind; others are years behind
  - Labels are less important than providing additional instruction where it is needed
  - RtI integrates regular and special education
    - Students with disabilities are in every tier

RtI Framework



What Is Not RtI?

- RtI is not a package
- RtI is neither tracking nor homogeneous grouping
  - RtI is *not* about providing different instruction to different groups of students, based on adult judgments about what students cannot do
- When it comes to mathematical thinking, any group of 2 or more students is heterogeneous
- And perhaps you have encountered students who seemed to be heterogeneous all by themselves

Effective Instructional Strategies (Tier 1)

- Problem-based learning
  - Rich problems can motivate concepts and skills
  - To learn problem solving, students must be given opportunities to solve (and struggle with) problems
- Differentiation *within* a task
  - Alternative to differentiation *by* task
  - Given a rich mathematical task, students differentiate themselves
  - Then teachers (and intervention specialists) provide whatever support students need (without giving too much away)

Effective Instructional Strategies (Tier 2)

- **What instructional strategies are effective in helping students with difficulties in mathematics?**
  - The use of structured peer-assisted learning activities
  - Systematic and explicit instruction using visual representations
  - Modifying instruction based on data from formative assessment of students (such as classroom discussions or quizzes)
  - Providing opportunities for students to think aloud while they work

Source: Research Brief from the National Council of Teachers of Mathematics. Available at http://www.nctm.org/news/content.aspx?id=8468

Key Messages from the CCSS for Mathematics

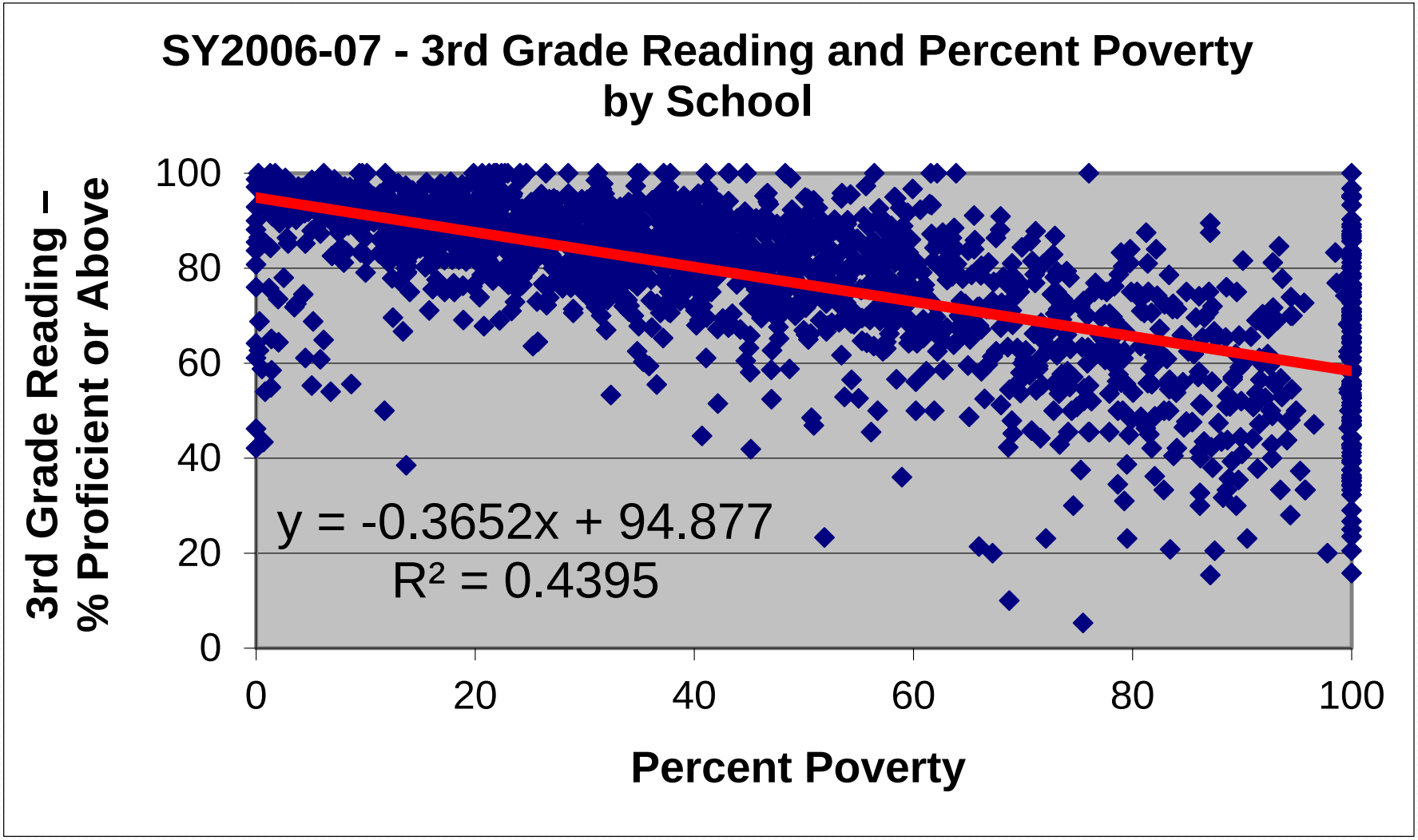
What's New with the CCSS?

- Common across 45+ states
- Internationally benchmarked standards
- Focus, coherence, and rigor
- College and career readiness for all
- And all students means ALL students

College and Career Readiness

- College and career readiness involves mathematics at the level of Algebra 2 or its equivalent (A2E)
- All students need proficiency in A2E for
  - Many careers, with or without college
  - Informed citizenship
  - Individual empowerment
- High school mathematics should open doors
  - But adult decisions often close doors for students
  - After students complete A2E, they have choices
- But not your parents' Algebra 2

Who Can Interpret This?



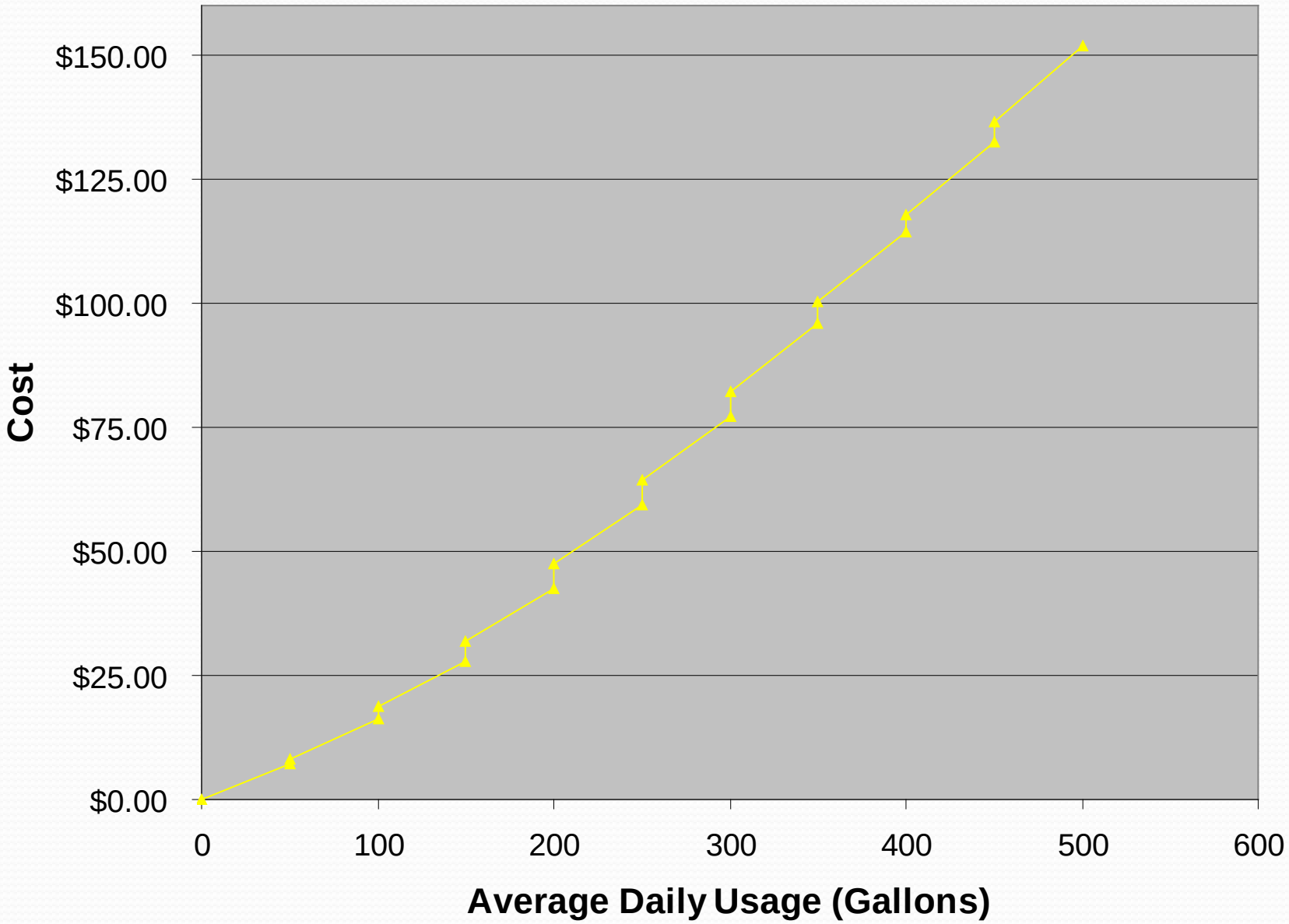
Washington Suburban Sanitary Commission

Rate Schedule, July 1, 2008

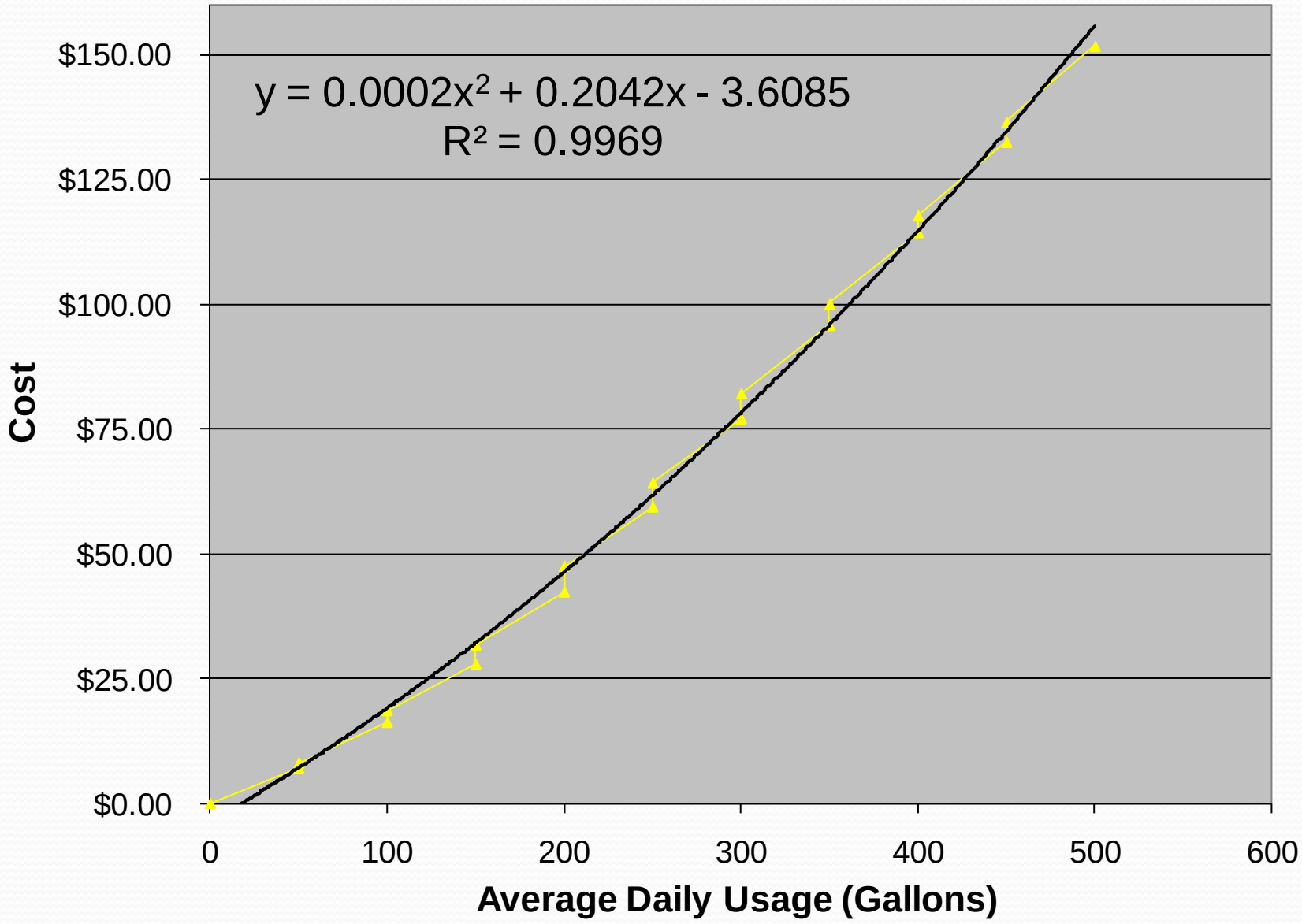
Average Daily Consumption (Gallons/Day)	Water Rate Per 1,000 Gallons	Sewer Rate Per 1,000 Gallons	Combined Rate Per 1,000 Gallons
0-49	$1.97	$2.77	$4.74
50 - 99	2.21	3.22	5.43
100 - 149	2.42	3.79	6.21
150 - 199	2.71	4.36	7.07
200 - 249	3.17	4.76	7.93
250 - 299	3.43	5.14	8.57
300 - 349	3.63	5.50	9.13
350 - 399	3.79	5.75	9.54
400 - 449	3.94	5.88	9.82
…	…	…	…

Source: http://www.wsscwater.com/service/rates.cfm

Monthly Water and Sewer Cost



Monthly Water and Sewer Cost



What Is Needed?

- Renewed curriculum and instruction
  - Especially across middle and high school, toward a rigorous, relevant, and accessible A2E
- Support for students are behind
  - To help them catch up

- The CCSS and the Model Pathways are foundational responses to these needs

A Look Inside the Common Core State Standards

CCSS Principles

- **Focus**: focus strongly on key ideas, understandings, and skills in each grade and course
- **Coherence**: think across grades, and link to major topics in each grade
- **Rigor**: in major topics, pursue with equal intensity
  - conceptual understanding,
  - procedural skill and fluency, and
  - applications

CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

K-8 CCSS Changing Content Emphases

- Primary focus on number in grades K-5
- Fractions as numbers on the number line, beginning with unit fractions
- Fluency with standard algorithms, supported by strategies based in place value
- Much statistics in grade 6-8
- Much algebra and geometry in grades 7-8

CCSS for High School Mathematics

- Organized in "Conceptual Categories"
  - Number and Quantity
  - Algebra
  - Functions
  - Modeling
  - Geometry
  - Statistics and Probability
- Conceptual categories are not courses
- Additional mathematics for advanced courses indicated by (+)
- Standards with connections to modeling indicated by (★)

Conceptual Category Introduction

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here

Conceptual Category Overview

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Domain

Making Inferences and Justifying Conclusions

Cluster

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Format of High School Standards

Domain

Cluster

Standard

Advanced

The Complex Number System **N-CN**

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number

High School Mathematics Today

- Algebra 1 and Geometry courses typically
  - Reteach much middle grades content
- Algebra 2 courses typically
  - Reteach Algebra 1
  - Include some statistics and probability
  - Include optional topics
  - Pre-teach Precalculus content
- ***Algebra 2 is two miles wide***
  - *And a quarter inch deep*

HS CCSS Changing Content Emphases

- Number and quantity
  - Number systems, attention to units
- Modeling
  - Threaded throughout the standards
- Geometry
  - Proof for all, based on transformations
- Algebra and functions
  - Organized by mathematical practices
- Statistics and probability
  - Inference for all, based on simulation

CCSS Domain Progression

<table>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting & Cardinality</td><td colspan="9"></td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td><td rowspan="2">Number & Quantity</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations – Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="9">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics & Probability</td></tr>
</table>

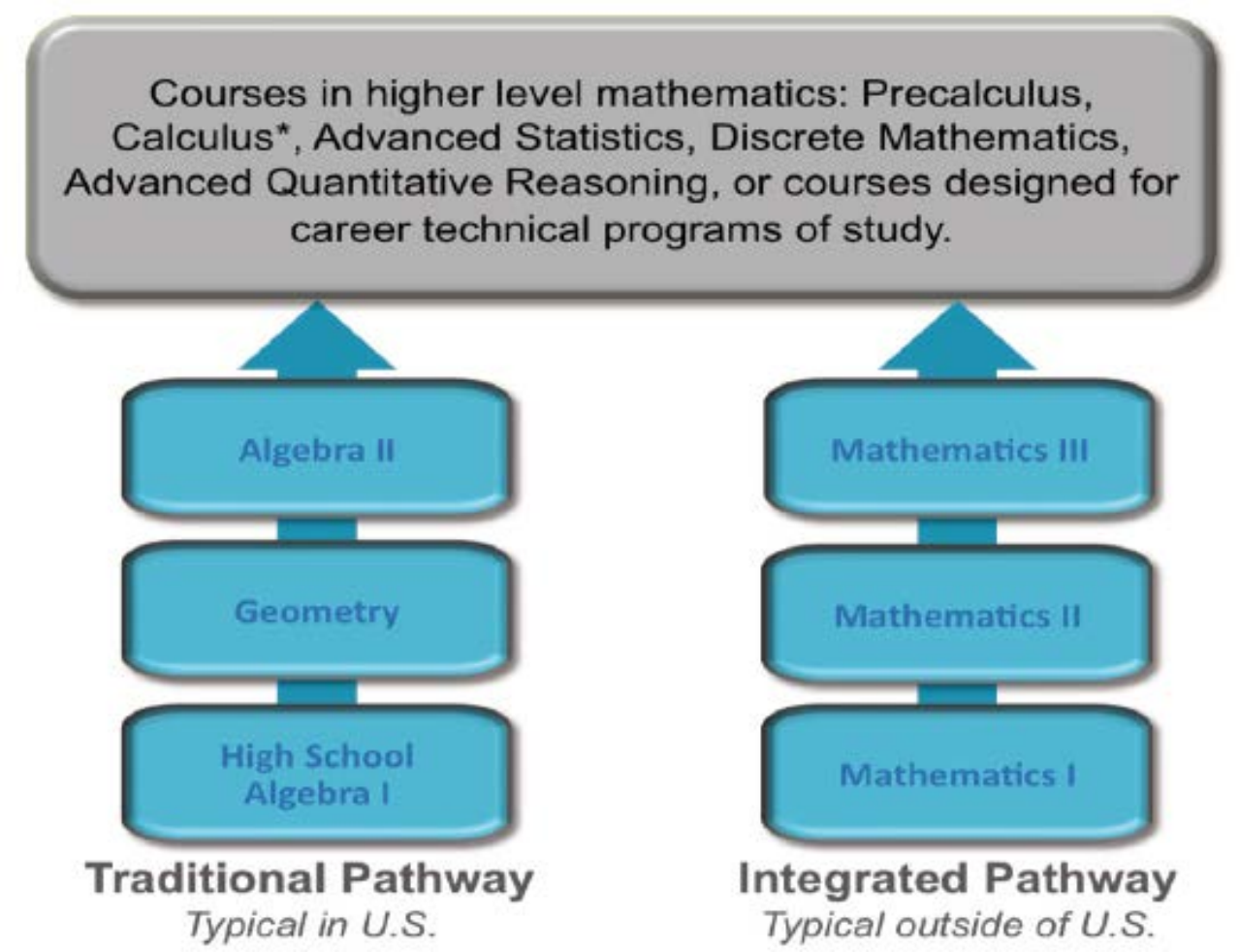
High School Mathematics Pathways

- CCSS Appendix A, developed by Achieve
- Two main pathways:
  - Traditional: Two algebra courses and a geometry course, with statistics and probability in each
  - Integrated: Three courses, each of which includes algebra, geometry, statistics, and probability
- Both pathways:
  - Complete the Core in the third year
  - Include the same "critical areas"
  - Require rethinking high school mathematics
  - Prepare students for a menu of fourth-year courses

Typical in U.S.

Typical outside U.S.

Two Main Pathways



Comparison of Pathways "Units"

Relationships Between Quantities Linear and Exponential Rel. Descriptive Statistics Expressions and Equations Quadratic Functions and Modeling	Relationships Between Quantities Linear and Exponential Rel. Reasoning with Equations Descriptive Statistics Congruence and Constructions Connecting A & G through Coords.
Congruence and Constructions Similarity and Trigonometry Extending to Three Dimensions Connecting A & G through Coords. Circles w/ and w/o Coordinates Applications of Probability	Extending the Number System Quadratic Functions and Modeling Expressions and Equations Applications of Probability Similarity and Trigonometry Circles w/ and w/o Coordinates
Polynomial, Rational, and Radical Rel. Trigonometric Functions Modeling with Functions Inferences and Conclusions from Data	Inferences and Conclusions from Data Polynomial, Rational, and Radical Rel. Trigonometric Functions Mathematical Modeling

Assessment and Program Considerations

Assessment Design

- Smarter Balanced Assessment Consortium (SBAC)
- Computer adaptive summative and formative assessments
  - Grade-level assessments in grades 3-8
  - High school assessment at grade 11
- **Innovative item types** go beyond multiple choice questions to include constructed response and performance tasks that measure critical thinking and problem solving.
- **Interim assessments** provide information about student progress throughout the year to help teachers differentiate instruction.
- A **digital library** of research-based formative assessment practices and tools.

SBAC Mathematics Claims

- Claim #1: **Concepts and Procedures (40%)**
  - Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency
- Claim #2: **Problem Solving (20%)**
  - Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies
- Claim #3: **Communicating Reasoning (20%)**
  - Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others
- Claim #4: **Modeling and Data Analysis (20%)**
  - Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems

Claim 1 Assessment Targets (Grade 11)

Number and Quantity

A. Extend the properties of exponents to rational exponents.
B. Use properties of rational and irrational numbers.
C. Reason quantitatively and use units to solve problems.

Algebra

D. Interpret the structure of expressions.
E. Write expressions in equivalent forms to solve problems.
F. Perform arithmetic operations on polynomials.
G. Create equations that describe numbers or relationships.
H. Understand solving equations as a process of reasoning and explain the reasoning.
I. Solve equations and inequalities in one variable.
J. Represent and solve equations and inequalities graphically.

Claim 1 Assessment Targets (Grade 11)

Functions

K. Understand the concept of a function and use function notation.
L. Interpret functions that arise in applications in terms of a context.
M. Analyze functions using different representations.
N. Build a function that models a relationship between two quantities.

Geometry

O. Prove geometric theorems.

Statistics and Probability

P. Summarize, represent and interpret data on a single count or measurement variable.

Claim 2 – Problem Solving

A. Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace

B. Select and use tools strategically

C. Interpret results in the context of the situation

D. Identify important quantities in a practical situation and map their relationships.

Claim 3 – Communicating Reasoning

A. Test propositions or conjectures with specific examples.
B. Construct, autonomously, chains of reasoning that justify or refute propositions or conjectures.
C. State logical assumptions being used.
D. Use the technique of breaking an argument into cases.
E. Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.
F. Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
G. Determine conditions under which an argument does and does not apply.

Claim 4 – Modeling and Data Analysis

A. Apply mathematics to solve problems arising in everyday life, society, and the workplace.
B. Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.
C. State logical assumptions being used.
D. Interpret results in the context of a situation.
E. Analyze the adequacy of and make improvement to an existing model or develop a mathematical model of a real phenomenon.
F. Identify important quantities in a practical situation and map their relationships.
G. Identify, analyze, and synthesize relevant external resources to pose or solve problems.

Prealgebra at High School?

- Prealgebra should not count as high school mathematics
  - Preparation for current HS graduation tests
  - College admissions requirements (and NCAA)
  - Reaching college and career readiness
- You can't help students catch up by slowing them down
- NCAA counts Algebra A and Algebra B as one credit
- When students are behind
  - Give them access to the regular curriculum *and* extra support (*a la* Response to Intervention)

CCSSM and Acceleration

- The CCSSM represent significant curricular acceleration in grades K-8
  - Much Algebra 1, Geometry, and Statistics are in the middle grades
  - Many "accelerated" programs will no longer be ahead
  - The CCSS for Grade 8 is a reasonable, internationally benchmarked response to "Algebra for all" in grade 8
- Accelerating large percentages of students much beyond the CCSS for K-8 is probably unwise
- The CCSSM for high school include much advanced content and much new content for all students
  - Most students will need three years in high school to complete CCSS
- *So we need to rethink mathematics, grades 6-12*

Implementation Resources

Implementation Resources

- The Mathematics Frameworks from the Partnership for Readiness for College and Careers (PARCC)
- The draft Mathematics Content Specifications from the Smarter Balanced Assessment Consortium (SBAC)
- The Mathematics Assessment Project (MAP)
- The Illustrative Mathematics Project (IMP)
- Bill McCallum's Common Core Tools blog
  - Progressions documents
- Common Core videos from the Hunt Institute
- Phil Daro's SERP Institute videos
- Inside Mathematics website

An Example from MAP

Boomerangs

Phil and Cath make and sell boomerangs for a school event.
The money they raise will go to charity.

They plan to make them in two sizes: small and large.

Phil will carve them from wood.
The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve.
Phil has a total of 24 hours available for carving.

Cath will decorate them.
She only has time to decorate 10 boomerangs of either size.

The small boomerang will make $8 for charity.
The large boomerang will make $10 for charity.
They want to make as much money for charity as they can.

How many small and large boomerangs should they make?

How much money will they then make?

Alex's solution

Phil can only make 12 small or 8 large boomerangs in 24 hours

12 small makes $96

8 large makes $80

Cath only has time to make 10, so $96 is impossible.

She could make 10 small boomerangs which will make $80.

So she either makes 8 large or 10 small boomerangs

and makes $80.

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Implementation Suggestions

For large and medium schools

Math Programs for All Students

- Main pathway completing the CCSS in grade 11
  - Rather than Prealgebra in grade 9, provide support for *all* students to reach these standards
  - Provide alternatives to Precalculus for seniors
- Alternative pathway completing the CCSS in grade 10, allowing for AP Calculus in grade 12
  - Determine where "compacting" should happen
- Flexibility for the small numbers of students who are eager for still more mathematics
  - Align with gifted education policies
  - Expect PSEO during senior year

For small schools

Math Programs for All Students

- Main pathway completing the CCSS in grade 11
  - Rather than Prealgebra in grade 9, Provide support for *all* students to reach these standards
  - Employ distance learning for seniors
- Flexibility for students who are eager for still more mathematics
  - Employ acceleration on a case-by-case basis, driven by interest, emphasizing depth of learning
  - Employ distance learning as appropriate
  - Attend to gifted education policies

Research-Based Principles

- Implementation matters
  - Variation within a model is greater than the variation between models
  - Adoption of standards, programs, or textbooks merely opens the door
- High-quality professional development
  - Focuses on the content the teachers are teaching
  - Draws on curricular materials teachers are using
  - Involves analyzing student work
  - Takes time

Maintain Focus and Coherence

- Implementation plans may miss the point
  - Readers might not see focus and coherence
  - Strategies may be counterproductive
- The goal is coherence in curriculum, instruction, and learning
  - Standards are taken as atoms, but the power is in the bonds (Jason Zimba)
  - Think in chapters, not lessons (Phil Daro)

Tips for Implementation

1. Get to know the CCSS
   - Use the critical areas of focus
   - Take a progressions view
2. Lead with the mathematical practices
   - With the content you are teaching now
3. Work collectively
   - You do not need to invent it all yourself
4. Involve administrators and parents
5. Take some transitional steps
   - Changes you can make soon

Tips for Implementation

6. Build support structures for students who are behind
7. Design programs for *all students*, driven by progressions, not course names
8. Require focus and coherence in district initiatives and professional development offerings
9. Document your implementation
   - Treat your implementation work as action research
10. Take a deep breath … and prepare for a long haul
    - Improving instruction and building new systems takes time

Questions

Implementation Questions for You

- Can we empower teachers to make necessary changes?
  - Curriculum, instruction, support, programs, …
- Can we get the incentives right?
  - So that teachers will regularly work together to reach more students more of the time
  - So that we all learn from and with our best teachers
- Can we bring mathematics leadership to the decision-making table?
  - So that school-improvement efforts focus on long-term improvements not short-term fixes