This Is Not Your Parents' High School Math Class

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North Olmsted City Schools

Underlying Principle

- "*Everyone is good at mathematics because everyone can think. And mathematics is about thinking.*"
  - Yeap Ban Har, National Institute of Education, Singapore.
- Corollary 1: Strategies that attempt to remove thinking from learning are bound to fail in the long run.
- Corollary 2: When learning is effective, "getting the right answer" is but a small piece of the work.

What's New with the CCSS?

- Common across 45+ states
- Internationally benchmarked standards
- Focus, coherence, and rigor
- College and career readiness for all
- And all students means ALL students

Major Themes

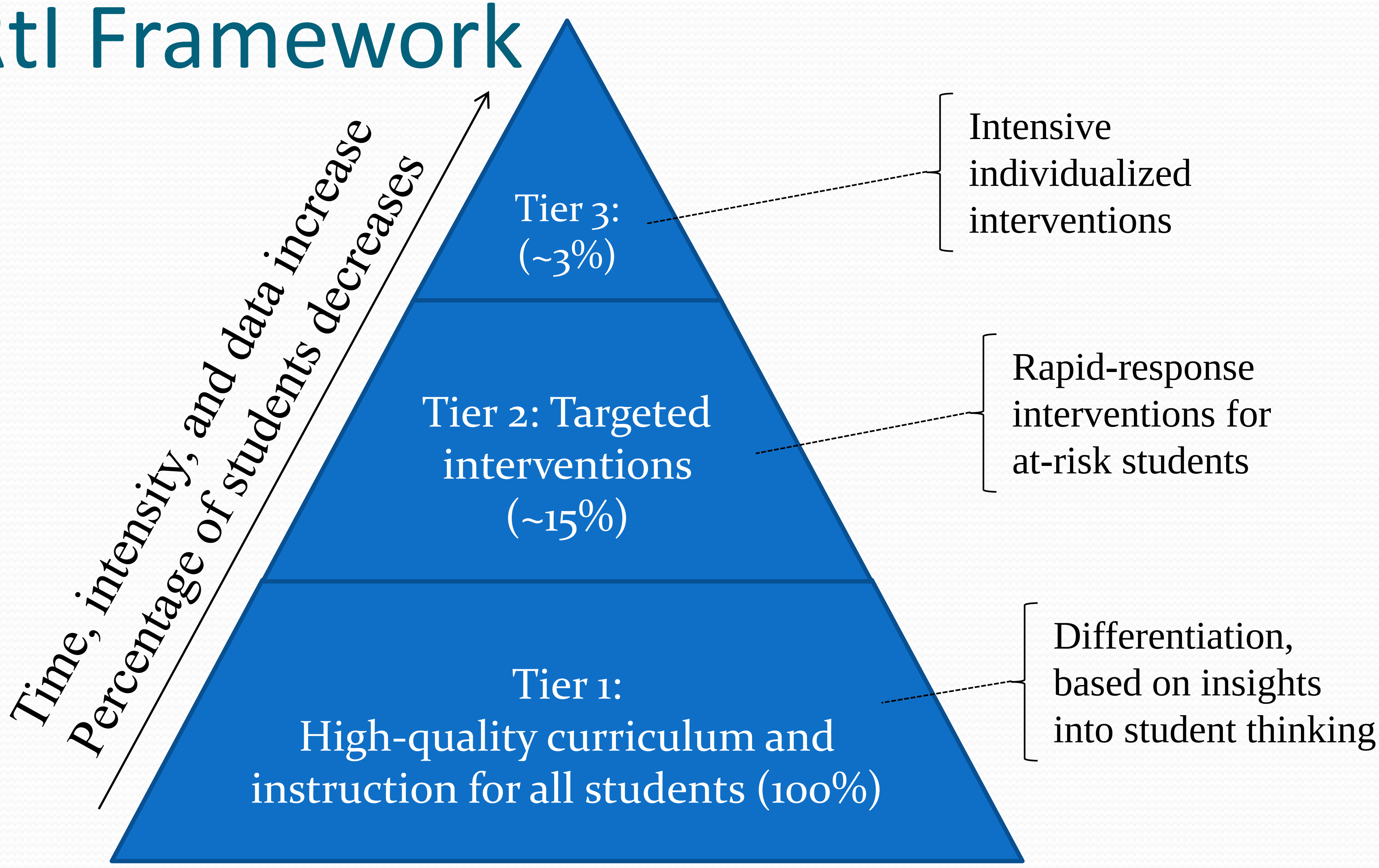
- All students means ALL students
- The work is about improving instruction, which requires that teachers (all teachers) collaborate to reach more students more of the time
- Common messages among current initiatives
  - Common Core State Standards
  - Formative Assessment
  - Response to Intervention
  - School Turnaround
  - …

Key Messages from Response to Intervention

What Is RtI?

- RtI is about establishing a school-wide system for allocating instructional resources where they are needed
  - Give all students access to the regular curriculum AND provide differentiated instruction and support
  - Some students are 15 minutes behind; others are years behind
  - Labels are less important than providing additional instruction where it is needed
  - RtI integrates regular and special education
    - Students with disabilities are in every tier

RtI Framework



What Is Not RtI?

- RtI is not a package
- RtI is neither tracking nor homogeneous grouping
  - RtI is *not* about providing different instruction to different groups of students, based on adult judgments about what students cannot do
- When it comes to mathematical thinking, any group of 2 or more students is heterogeneous
- And perhaps you have encountered students who seemed to be heterogeneous all by themselves

Effective Instructional Strategies (Tier 1)

- Problem-based learning
  - Rich problems can motivate concepts and skills
  - To learn problem solving, students must be given opportunities to solve (and struggle with) problems
- Differentiation *within* a task
  - Alternative to differentiation *by* task
  - Given a rich mathematical task, students differentiate themselves
  - Then teachers (and intervention specialists) provide whatever support students need (without giving too much away)

Effective Instructional Strategies (Tier 2)

- **What instructional strategies are effective in helping students with difficulties in mathematics?**
  - The use of structured peer-assisted learning activities
  - Systematic and explicit instruction using visual representations
  - Modifying instruction based on data from formative assessment of students (such as classroom discussions or quizzes)
  - Providing opportunities for students to think aloud while they work

Source: Research Brief from the National Council of Teachers of Mathematics. Available at http://www.nctm.org/news/content.aspx?id=8468

Essential Shifts for the CCSS for Mathematics

Instructional Shifts

- Focus strongly where the Standards focus
- Coherence: think across grades, and link to major topics within grades
- Rigor: require conceptual understanding, procedural skill and fluency, and application with equal intensity

Assessment Shifts: PARCC Task Types

TYPE I: *TASKS ASSESSING CONCEPTS, SKILLS AND PROCEDURES*	TYPE II: *TASKS ASSESSING EXPRESSING MATHEMATICAL REASONING*	TYPE III: *TASKS ASSESSING MODELING / APPLICATIONS*
• A balance of conceptual understanding, fluency, and application • Any or all mathematical practice standards • Machine scorable, innovative, computer-based formats • Included on the End of Year and Performance Based Assessment	• Written arguments, justifications, critique of reasoning, precision in mathematical statements • MP.3, MP.6 and other mathematical practice standards • A mix of innovative, machine scored and hand scored responses • Included on the Performance Based Assessment	• Modeling and application in a real-world context or scenario • MP.4 and other mathematical practice standards • A mix of innovative, machine scored and hand scored responses • Included on the Performance Based Assessment

CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Implications

- The PARCC assessments take seriously the Standards for Mathematical Practices
- Success on these new assessments will require shifts in instruction, in classroom assessment, and in the nature of the content:
  - Correct answers are insufficient
  - Explanation and reasoning are required
  - Modeling and application are crucial

K-8 Content Shifts

- Primary focus on number in grades K-5
- Fluency with standard algorithms, supported by strategies based in place value
- Fractions as numbers on the number line, built from unit fractions
  - Unit fractions provide meaning for fraction arithmetic
- Much statistics in grade 6-8
- Much algebra and geometry in grades 7-8
  - Fractions $\Rightarrow$ Proportional reasoning $\Rightarrow$ Linear functions

High School Content Shifts

- Number and quantity
  - Number systems, attention to units
- Modeling
  - Threaded throughout the standards
- Geometry
  - Proof for all, based on transformations
- Algebra and functions
  - Organized by mathematical practices
- Statistics and probability
  - Inference for all, based on simulation

Programmatic Shifts

- The CCSSM represent significant curricular acceleration in grades K-8
  - Much Algebra 1, Geometry, and Statistics are in the middle grades
  - Many "accelerated" programs will no longer be ahead
  - The CCSS for Grade 8 is a reasonable, internationally benchmarked response to "Algebra for all" in grade 8
- Accelerating large percentages of students much beyond the CCSS for K-8 is probably unwise
- The CCSSM for high school include much advanced content and much new content for all students
  - Most students will need three years in high school to complete CCSS
- *So we need to rethink mathematics, grades 6-12*

Math Programs for All Students

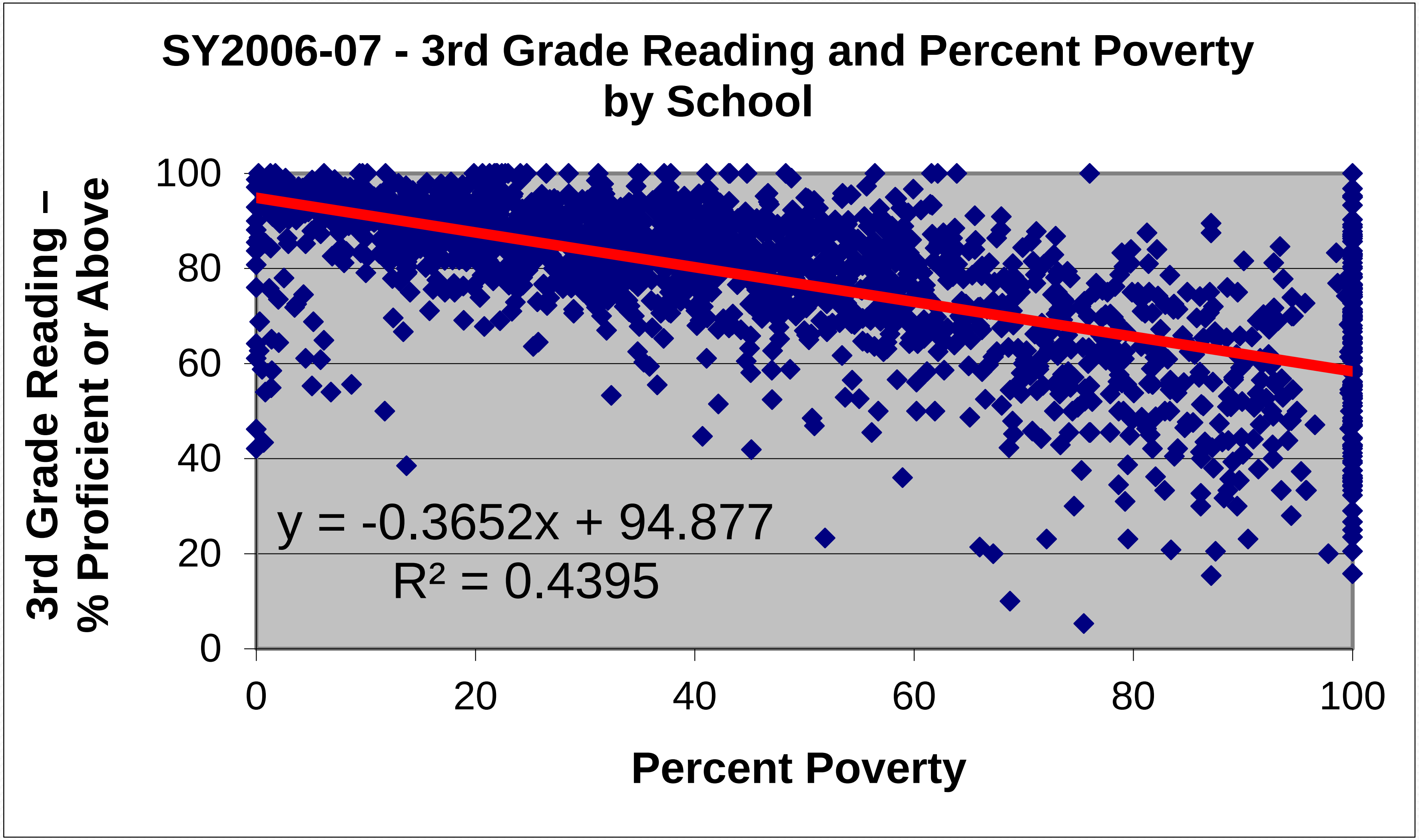
- Main pathway completing the CCSS in grade 11
  - Rather than Prealgebra in grade 9, provide support for *all* students to reach these standards
  - Provide alternatives to Precalculus for seniors
- Alternative pathway completing the CCSS in grade 10, allowing for AP Calculus in grade 12
  - Determine where "compacting" should happen
- Flexibility for the small numbers of students who are eager for still more mathematics
  - Align with gifted education policies
  - Expect PSEO during senior year

Mathematics for College and Career Readiness

College and Career Readiness

- College and career readiness involves mathematics at the level of Algebra 2 or its equivalent (A2E)
- All students need proficiency in A2E for
  - Many careers, with or without college
  - Informed citizenship
  - Individual empowerment
- High school mathematics should open doors
  - But adult decisions often close doors for students
  - After students complete A2E, they have choices
- But not your parents' Algebra 2

Who Can Interpret This?



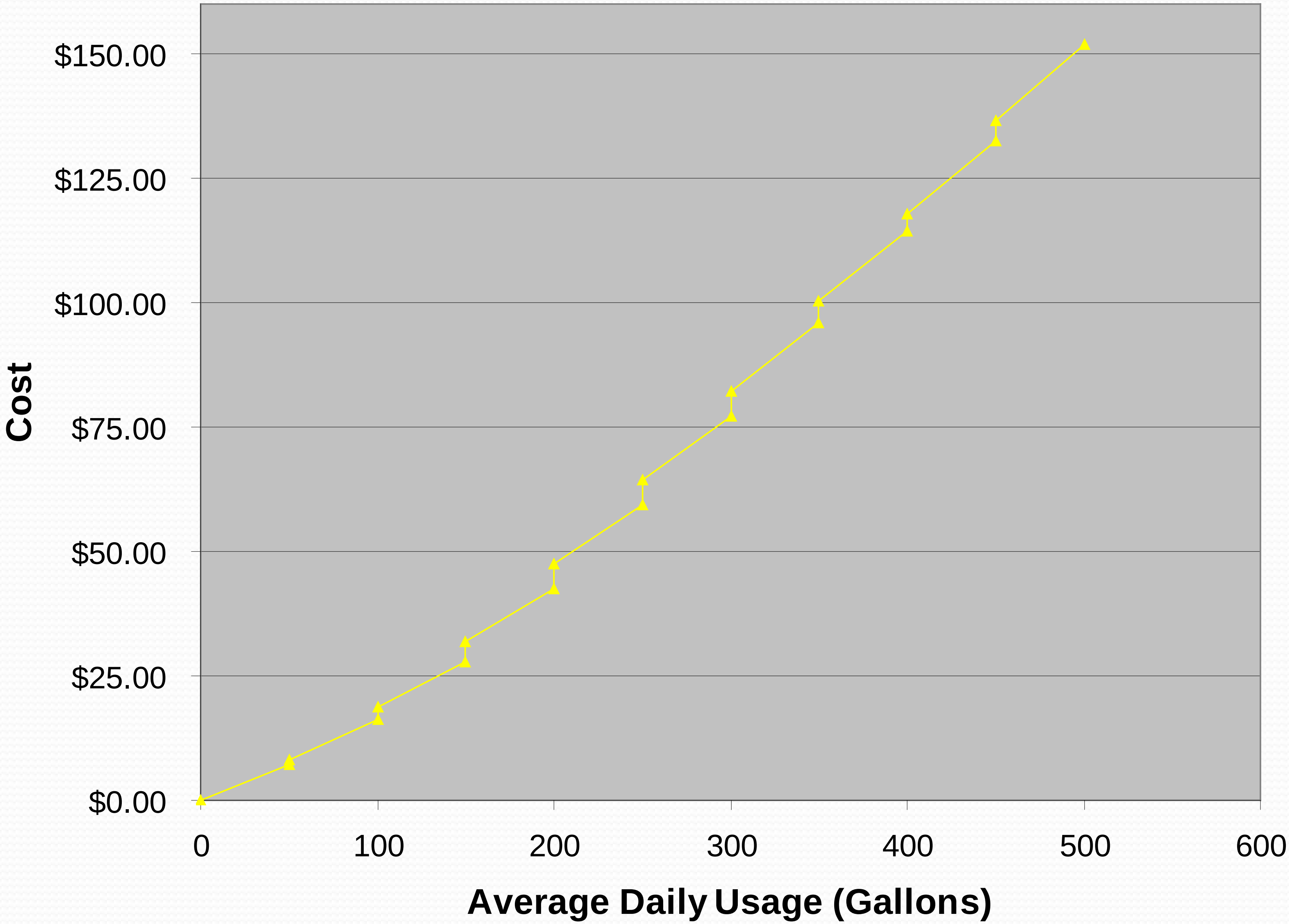
Washington Suburban Sanitary Commission

Rate Schedule, July 1, 2008

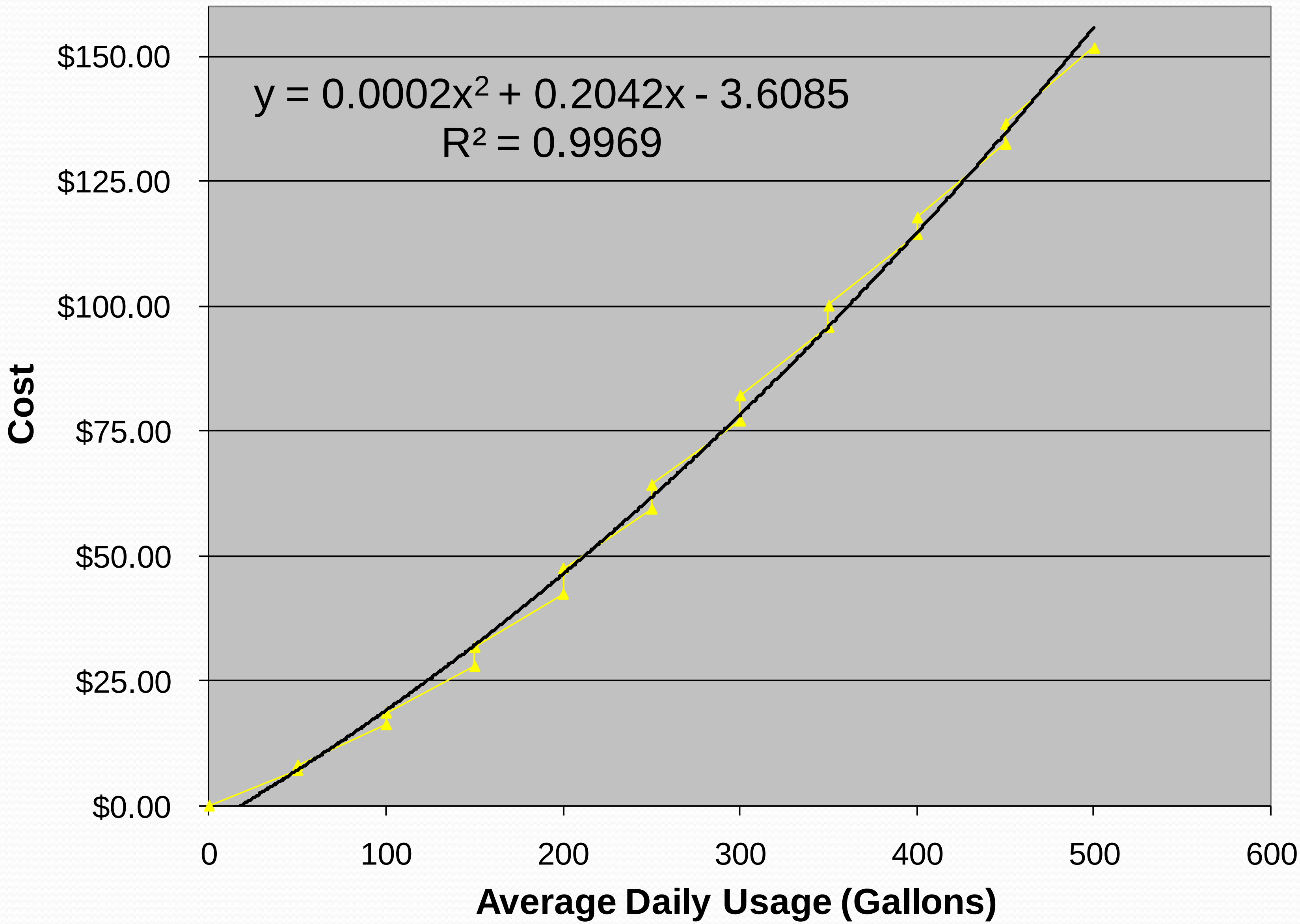
Average Daily Consumption (Gallons/Day)	Water Rate Per 1,000 Gallons	Sewer Rate Per 1,000 Gallons	Combined Rate Per 1,000 Gallons
0-49	$1.97	$2.77	$4.74
50 - 99	2.21	3.22	5.43
100 - 149	2.42	3.79	6.21
150 - 199	2.71	4.36	7.07
200 - 249	3.17	4.76	7.93
250 - 299	3.43	5.14	8.57
300 - 349	3.63	5.50	9.13
350 - 399	3.79	5.75	9.54
400 - 449	3.94	5.88	9.82
…	…	…	…

Source: http://www.wsscwater.com/service/rates.cfm

Monthly Water and Sewer Cost



Monthly Water and Sewer Cost



A Real-World Problem

- One weekend, I was gathering sticks from my lawn, bundling them in each hand. When both hands became full, I found that by using both hands for a single bundle, I could gather quite a few more sticks. Why? What relationship should I expect between the quantities of sticks gatherable by the two methods?

Sketch of Solution

- The capacity to gather sticks depends on the cross-sectional area of the circles made by my fingers. That area varies with the square of circumference. So two hands together have four times the capacity of one hand, or two times the capacity of both hands separately.
- *Note: With algebra, we can prove this more precisely.*

What Is Needed?

- Renewed curriculum and instruction
  - Especially across middle and high school, toward a rigorous, relevant, and accessible A2E
- Support for students are behind
  - To help them catch up

- The CCSS and the Model Pathways are foundational responses to these needs

High School Pathways

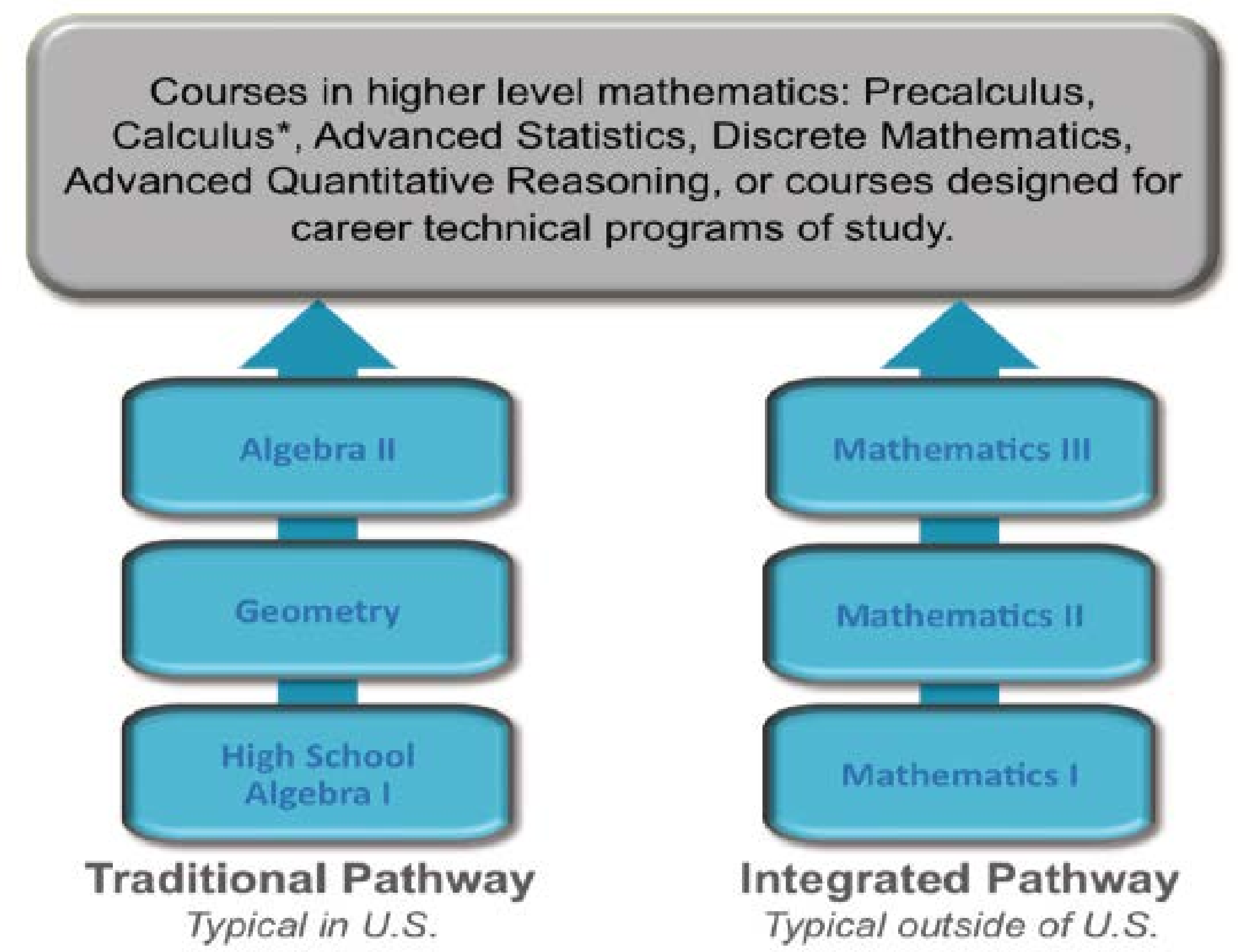
High School Mathematics Pathways

- CCSS Appendix A, developed by Achieve
- Two main pathways:
  - Traditional: Two algebra courses and a geometry course, with statistics and probability in each
  - Integrated: Three courses, each of which includes algebra, geometry, statistics, and probability
- Both pathways:
  - Complete the Core in the third year
  - Include the same "critical areas"
  - Require rethinking high school mathematics
  - Prepare students for a menu of fourth-year courses

Typical in U.S.

Typical outside U.S.

Two Main Pathways



Comparison of Pathways "Units"

Relationships Between Quantities Linear and Exponential Rel. Descriptive Statistics Expressions and Equations Quadratic Functions and Modeling	Relationships Between Quantities Linear and Exponential Rel. Reasoning with Equations Descriptive Statistics Congruence and Constructions Connecting A & G through Coords.
Congruence and Constructions Similarity and Trigonometry Extending to Three Dimensions Connecting A & G through Coords. Circles w/ and w/o Coordinates Applications of Probability	Extending the Number System Quadratic Functions and Modeling Expressions and Equations Applications of Probability Similarity and Trigonometry Circles w/ and w/o Coordinates
Polynomial, Rational, and Radical Rel. Trigonometric Functions Modeling with Functions Inferences and Conclusions from Data	Inferences and Conclusions from Data Polynomial, Rational, and Radical Rel. Trigonometric Functions Mathematical Modeling

Common Core Assessments

- The Partnership for Assessment of Readiness for College and Careers (PARCC) used these Pathways as the starting point for the design of end-of-course exams for these high school courses:

- Algebra 1, Geometry, Algebra 2

  and
- Mathematics 1, Mathematics 2, Mathematics 3

PARCC Mathematics Frameworks

- Individual end-of-course overviews
- For each course
  - Examples of key advances from previous grades
  - Discussion of Mathematical Practices in relation to course content
  - Fluency recommendations
- Pathway summary table
- Assessment limits table for standards assessed on more than one end-of-course test

Qualitative Shifts in Content

Pythagorean Theorem

- 8.G.6. Explain a proof of the Pythagorean theorem and its converse.
- 8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- G-SRT.4. Prove theorems about triangles. *Theorems include … the Pythagorean Theorem proved using triangle similarity.*
- G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem …
- F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

- *The Pythagorean Theorem is not just "$a^2 + b^2 = c^2$."*

Sequences as Functions

- F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
- F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- *Emphasize connections among patterns, sequences, and functions.*

Rules of Exponents

- 8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.
- N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- *These ideas support many HS standards on exponential functions.*

Solving Equations

- 8.EE.7.a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- A-REI.A. Understand solving equations as a process of reasoning and explain the reasoning
- A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;

- *This last standard supports the many different techniques for solving different types of equations.*

Seeing Structure in Expressions

- A-SSE.1. Interpret expressions that represent a quantity in terms of its context.
- A-SSE.2. Use the structure of an expression to identify ways to rewrite it.
- A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- *"Simplest form" depends on the purpose.*

Rational and Irrational Numbers

- 7.NS.2.d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 8.NS.1. Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.
- N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

- *This content is seldom taught.*

Can You Provide Three Explanations?

- Why is division by 0 undefined? Is $0/0 = 1$?
- Why is $a^0 = 1$? And does it matter what a is?
- Why is $a^{-n} = 1/a^n$? And does it matter what a is?
- Is 0 even, odd, neither, or both?
- Why is a negative times a negative positive?
- When multiplying fractions, why do we multiply numerators and denominators?
- When dividing by a fraction, why is it okay to invert and multiply?
- Is $0.9999... = 1$?

Implementation Resources and Suggestions

Implementation Resources

- The Mathematics Frameworks from the Partnership for Readiness for College and Careers (PARCC)
- The draft Mathematics Content Specifications from the Smarter Balanced Assessment Consortium (SBAC)
- The Mathematics Assessment Project (MAP)
- The Illustrative Mathematics Project (IMP)
- Bill McCallum's Common Core Tools blog
  - Progressions documents
- Common Core videos from the Hunt Institute
- Phil Daro's SERP Institute videos
- Inside Mathematics website

An Example from MAP

Boomerangs

Phil and Cath make and sell boomerangs for a school event.
The money they raise will go to charity.

They plan to make them in two sizes: small and large.

Phil will carve them from wood.
The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve.
Phil has a total of 24 hours available for carving.

Cath will decorate them.
She only has time to decorate 10 boomerangs of either size.

The small boomerang will make $8 for charity.
The large boomerang will make $10 for charity.
They want to make as much money for charity as they can.

How many small and large boomerangs should they make?

How much money will they then make?

Alex's solution

Phil can only make 12 small or 8 large boomerangs in 24 hours

12 small makes $96

8 large makes $80

Cath only has time to make 10, so $96 is impossible.

She could make 10 small boomerangs which will make $80.

So she either makes 8 large or 10 small boomerangs

and makes $80.

Tips for Implementation

1. Get to know the CCSS
   - Use the critical areas of focus
   - Take a progressions view
2. Lead with the mathematical practices
   - With the content you are teaching now
3. Work collectively
   - You do not need to invent it all yourself
4. Involve administrators and parents
5. Take some transitional steps
   - Changes you can make soon

Tips for Implementation

6. Build support structures for students who are behind
7. Design programs for *all students*, driven by progressions, not course names
8. Require focus and coherence in district initiatives and professional development offerings
9. Document your implementation
   - Treat your implementation work as action research
10. Take a deep breath … and prepare for a long haul
    - Improving instruction and building new systems takes time