Revising the Common Core State Standards for Mathematics: Challenges and opportunities in Ohio

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History of Common Core State Standards

- 2007: Governors and state superintendents began discussing shared standards across interested states
- 2009: Common Core State Standards Initiative launched
  - 48 states participated in the development of standards in ELA and mathematics
  - Broad, bipartisan effort
- 2010: Common Core State Standards (CCSS) released
  - Ohio adopted the Common Core in June 2010 without modification
  - By late 2011, Common Core adopted by 45.5 states and the District of Columbia
  - A few states later dropped CCSS; other states “customized” CCSS

Ohio's Standards Revision Infrastructure

- Public comment
  - Organized by standard
  - Called for claims, resolutions, and research/rationale
- Advisory Committee
  - Representatives of stakeholder organizations
  - Findell represented the Ohio Department of Higher Education
  - Reviewed public comments, provided directives for working groups
- Working Groups
  - Grade bands: K-5, 5-9, High School
- Ohio Department of Education staff

Ohio's Standards Revision Timeline, 2016

- March: Standards posted for public comment
- April and May: Advisory Committee
- May and June: Working Groups
- July: Proposed revisions posted for public comment
- August and September: Working Groups and Advisory Committee
- October to December: Presentation to Legislature and State Board of Education

Revision Process, by the Numbers

- 385 mathematics standards reviewed online
- 242 standards received 647 public comments
- 100 standards and 10 additional directives forwarded to Working Groups
- 155 changes to standards
  - Most changes were about clarity, embedding footnotes, or “clarity by HS course”
  - 13 standards with proposed revisions reverted to original
  - 41 standards with substantive changes, deleted, moved, or new
- 23 changes to cluster headings
  - 14 changes for clarity, including 7 embedded footnotes
  - 9 changes for vertical alignment, including 3 new clusters

Examples of Changes for Clarity

Clarifying Learning Targets

Original Standards	Revised Standards
Know number names and the count sequence. **K.CC.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	**Know number names and the count sequence.** **K.CC.2** Count forward within 100 beginning from any given number other than 1.
Know number names and the count sequence. **K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	**Know number names and the count sequence.** **K.CC.3** Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
K.CC.6 Compare numbers. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.	**Compare numbers.** **K.CC.6** Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.

Providing Helpful Detail

Original Standards	Revised Standards
Understand place value. **2.NBT.2** Count within 1000; skip-count by 5s, 10s, and 100s.	**Understand place value.** **2.NBT.2** Count forward and backward within 1000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5.
Understand place value. **2.NBT.3** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**Understand place value.** **2.NBT.3** Read and write numbers to 1000 using base-ten numerals, number names, expanded form, and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc.

Providing Cautionary Detail

Original Standards	Revised Standards
Develop understanding of fractions as numbers. **3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **a.** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. **b.** Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.	**Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.** **3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **a.** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. **b.** Represent a fraction *a/b* (which may be greater than 1) on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

Providing Too Much Detail?

Original Standards	Revised Standards
Perform operations with multi-digit whole numbers and with decimals to hundredths. **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.7** Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. **a.** Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place). **b.** Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). **c.** Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). *For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.*

Fending off Misinterpretation

Original Standards	Revised Standards
Represent and solve problems involving multiplication and division. **3.OA.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	**Represent and solve problems involving multiplication and division.** **3.OA.1** Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that *a* x *b* means *a* groups of *b* objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each).

Conforming to Convention

Original Standards	Revised Standards
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **5.MD.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative Property of Multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Transforming Without Coordinates

Original Standards	Revised Standards
Understand congruence and similarity using physical models, transparencies, or geometry software. **8.G.1** Verify experimentally the properties of rotations, reflections, and translations: **a.** Lines are taken to lines, and line segments to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines.	**Understand congruence and similarity using physical models, transparencies, or geometry software.** **8.G.1** Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **a.** Lines are taken to lines, and line segments are taken to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines.
Understand congruence and similarity using physical models, transparencies, or geometry software. **8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**Understand congruence and similarity using physical models, transparencies, or geometry software.** **8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.)

Broadening Views of Proof

Original Standards	Revised Standards
Understand and apply the Pythagorean Theorem. **8.G.6** Explain a proof of the Pythagorean Theorem and its converse.	**Understand and apply the Pythagorean Theorem.** **8.G.6** Analyze and justify an informal proof of the Pythagorean Theorem and its converse.
Prove geometric theorems. **G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	**Prove geometric theorems both formally and informally using a variety of methods.** **G.CO.9** Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
Prove theorems involving similarity. **G.SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	**Prove and apply theorems both formally and informally involving similarity using a variety of methods.** **G.SRT.4** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

Being Explicit About Intended Proofs

Original Standards	Revised Standards
Understand and apply theorems about circles. **G.C.1** Prove that all circles are similar.	**Understand and apply theorems about circles.** **G.C.1** Prove that all circles are similar using transformational arguments.
Understand and apply theorems about circles. **G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	**Understand and apply theorems about circles.** **G.C.2** Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
Understand and apply theorems about circles. **G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	**Understand and apply theorems about circles.** **G.C.3** Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.

Justifying and Verifying Sometimes

Original Standards	Revised Standards
Use coordinates to prove simple geometric theorems algebraically. **G.GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.** **G.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.
Solve systems of equations. **A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**Solve systems of equations.** **A.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Using More Friendly Language

Original Standards	Revised Standards
Use properties of operations to generate equivalent expressions. **7.EE.2** Understand that rewriting an expression in different forms in a problem context can shed light on a problem and how its quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*	**Use properties of operations to generate equivalent expressions.** **7.EE.2** In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p – 0.15p) is equivalent to (1 – 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.*
Understand the relationship between zeros and factors of polynomials. **A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Understand the relationship between zeros and factors of polynomials.** **A.APR.3** Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
Perform arithmetic operations with complex numbers. **N.CN.3** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	**Perform arithmetic operations with complex numbers.** **N.CN.3** (+) Find the conjugate of a complex number; use conjugates to find magnitudes and quotients of complex numbers.

Clarifying by High School Course

Original Standards	Revised Standards
Analyze functions using different representations. **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **a.** Graph linear and quadratic functions and show intercepts, maxima, and minima. **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **d.** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**Analyze functions using different representations.** **F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.* **a.** Graph linear functions and indicate intercepts. (A1, M1) **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) **c.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3) **d.** Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3) **e.** Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) **f.** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (A2, M3) **g.** (+) Graph rational functions, identifying zeros and asymptotes, when factoring is reasonable, and indicating end behavior. **h.** (+) Graph logarithmic functions, indicating intercepts and end behavior.

Examples of Substantive Changes

New Progression on Money

Original Standards	Revised Standards
Count to tell the number of objects. **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality. [a,b,c]	**Count to tell the number of objects.** **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies. [a,b,c]
Tell and write time. **1.MD.3** Tell and write time in hours and half-hours using analog and digital clocks.	**Work with time and money.** **1.MD.3** Work with time and money. **a.** Tell and write time in hours and half-hours using analog and digital clocks. **b.** Identify pennies and dimes by name and value.
Work with time and money. **2.MD.8** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	**Work with time and money.** **2.MD.8** Solve problems with money. **a.** Identify nickels and quarters by name and value. **b.** Find the value of a collection of quarters, dimes, nickels, and pennies. **c.** Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbols appropriately (not including decimal notation).

New Progression on Money (cont.)

Original Standards	Revised Standards
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	**Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.** **3.MD.1** Work with time and money. **a.** Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. **b.** Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbol appropriately (not including decimal notation).
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. **4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** **4.MD.2** Solve real-world problems involving money, time, and metric measurement. **a.** Using models, add and subtract money and express the answer in decimal notation. **b.** Using number line diagrams, clocks, or other models, add and subtract intervals of time in hours and minutes. **c.** Add, subtract, and multiply whole numbers to solve metric

Measurement Separation

Original Standards	Revised Standards
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. **4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),...*	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** **4.MD.1** Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. *For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...*
Convert like measurement units within a given measurement system. **5.MD.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	**Convert like measurement units within a given measurement system.** **5.MD.1** Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.

Order of Operations Postponed to Grade 6

Original Standards	Revised Standards
Solve problems involving the four operations, and identify and explain patterns in arithmetic. **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.** **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.
Write and interpret numerical expressions. **5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	**Write and interpret numerical expressions.** **5.OA.1** Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary.

Order of Operations

Original Standards	Revised Standards
Apply and extend previous understandings of arithmetic to algebraic expressions. **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers. **a.** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* **b.** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*	**Apply and extend previous understandings of arithmetic to algebraic expressions.** **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers. **a.** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* **b.** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*

Geometry Progression

Polygons and Properties

Original Standards	Revised Standards
Reason with shapes and their attributes. **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	**Reason with shapes and their attributes.** **3.G.1** Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).
Classify two-dimensional figures into categories based on their properties. **5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	**Classify two-dimensional figures into categories based on their properties.** **5.G.3** Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles).
Classify two-dimensional figures into categories based on their properties. **5.G.4** Classify two-dimensional figures in a hierarchy based on properties.	**Classify two-dimensional figures into categories based on their properties.** **5.G.4** Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids, and rhombuses.

Polygons and Properties (cont.)

Original Standards	Revised Standards
Draw, construct, and describe geometrical figures and describe the relationships between them. **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	**Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.
New Standard	**Classify and analyze geometric figures.** **G.CO.14** Classify two-dimensional figures in a hierarchy based on properties.

Symmetry

Original Standards	Revised Standards
Draw and identify lines and angles, and classify shapes by properties of their lines and angles. **4.G.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Deleted Standard
Experiment with transformations in the plane. **G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	**Experiment with transformations in the plane.** **G.CO.3** Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. **b.** Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.

Circles

Original Standards	Revised Standards
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. **7.G.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.
Find arc lengths and areas of sectors of circles. **G.C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	**Find arc lengths and areas of sectors of circles.** **G.C.5** Find arc lengths and areas of sectors of circles. **a.** Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. (G, M2) **b.** Derive the formula for the area of a sector, and use it to solve problems. (G, M2)
New Standard	**Find arc lengths and areas of sectors of circles.** **G.C.6** Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)

Similarity

Original Standards	Revised Standards
Draw, construct, and describe geometrical figures and describe the relationships between them. **7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. **b.** Represent proportional relationships within and between similar figures.
New Standard	**Understand the relationships between lengths, area, and volumes.** **G.GMD.5** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.
New Standard	**Understand the relationships between lengths, area, and volumes.** **G.GMD.6** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.

Statistics in Grades 6 and 7

GAISE Model in Grade 6

Original Standards	Revised Standards
Develop understanding of statistical variability. **6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**Develop understanding of statistical problem solving.** **6.SP.1** Develop statistical reasoning by using the GAISE model: **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)

Reiterate GAISE Model in Grade 7

Original Standards	Revised Standards
Use random sampling to draw inferences about a population. **7.SP.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	**Broaden understanding of statistical problem solving.** **7.SP.2** Broaden statistical reasoning by using the GAISE model. **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?"* (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)

Remove MAD from Grade 6

Original Standards	Revised Standards
Summarize and describe distributions. **6.SP.5** Summarize numerical data sets in relation to their context, such as by: **a.** Reporting the number of observations. **b.** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **d.** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**Summarize and describe distributions.** **6.SP.5** Summarize numerical data sets in relation to their context. **a.** Report the number of observations. **b.** Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. **d.** Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.

Include MAD in Grade 7

Original Standards	Revised Standards
Draw informal comparative inferences about two populations. **7.SP.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	**Summarize and describe distributions representing one population and draw informal comparisons between two populations.** **7.SP.3** Describe and analyze distributions. **a.** Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point. **b.** Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable.*

Move Sampling and Inference to HS

Original Standards	Revised Standards
Use random sampling to draw inferences about a population. **7.SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	**Use sampling to draw conclusions about a population.** **7.SP. 1** Understand that statistics can be used to gain information about a population by examining a sample of the population. **a.** Differentiate between a sample and a population. **b.** Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.
Draw informal comparative inferences about two populations. **7.SP.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	Deleted Standard

Other Content Adjustments

Inverses of Functions

Original Standards	Revised Standards
Build new functions from existing functions. **F.BF.4** Find inverse functions. **a.** Solve an equation of the form *f(x) = x* for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2x³ or f(x) = (x+1)/(x–1) for x ≠ 1.* **b.** (+) Verify by composition that one function is the inverse of another. **c.** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. **d.** (+) Produce an invertible function from a non-invertible function by restricting the domain.	**Build new functions from existing functions.** **F.BF.4** Find inverse functions. **a.** Informally determine the input of a function when the output is known. (A1, M1) **b.** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3) **c.** (+) Verify by composition that one function is the inverse of another. (A2, M3) **d.** (+) Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3) **e.** (+) Produce an invertible function from a non-invertible function by restricting the domain.

Standards Partly Removed

Original Standards	Revised Standards
Know that there are numbers that are not rational, and approximate them by rational numbers. **8.NS.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**Know that there are numbers that are not rational, and approximate them by rational numbers.** **8.NS.1** Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.

Standards Changed to (+)

Original Standards	Revised Standards
Translate between the geometric description and the equation for a conic section. **G.GPE.2** Derive the equation of a parabola given a focus and directrix.	**Translate between the geometric description and the equation for a conic section.** **G.GPE.2** (+) Derive the equation of a parabola given a focus and directrix.
Write expressions in equivalent forms to solve problems. **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**	**Write expressions in equivalent forms to solve problems.** **A.SSE.4** (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

Standards Partly Changed to (+)

Original Standards	Revised Standards
Define trigonometric ratios and solve problems involving right triangles. **G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*	**Define trigonometric ratios and solve problems involving right triangles.** **G.SRT.8** Solve problems involving right triangles.* **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2) **b.** (+) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* (Alg 2, M3)
Solve equations and inequalities in one variable. **A.REI.4** Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **b.** Solve quadratic equations by inspection e.g., for $x^2 = 49$, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	**Solve equations and inequalities in one variable.** **A.REI.4** Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (+) **c.** Derive the quadratic formula using the method of completing the square.

Draft Revisions Reconsidered

A Window into the Process

Missing the Point of a Standard

Original Standards	Draft Revisions Reconsidered
HSA.REI.11 Represent and solve equations and inequalities graphically. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	**Represent and solve equations and inequalities graphically.** **A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equation *y=f(x)* and *y=g(x)* intersect are the solutions of the equation *f(x)=g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. a. Focus on cases where *f(x)* and/or *g(x)* are linear or exponential functions. (A1, M1) b. Include cases where *f(x)* and/or *g(x)* are combinations of linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (A2, M3)

Exacerbating a Puzzling Standard

Original Standards	Draft Revisions Reconsidered
HSF.BF.4 Build new functions from existing functions. Find inverse functions. a. Solve an equation of the form *f(x)* = *x* for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	**Build new functions from existing functions.** **F.BF.4** Find inverse functions. a. Find the inverse of a function *f(x)* algebraically. *For example, if f(x)=3x, then the inverse of f(x) is x/3.* b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

Separating Procedures from Concepts

Original Standards	Draft Revisions Reconsidered
HSA.SSE.4 Write expressions in equivalent forms to solve problems. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*	**Write expressions in equivalent forms to solve problems.** **A.SSE.4** Solve real-world and mathematical problems involving the sum of a finite geometric series.* a. Use the formula to solve problems. *For example, calculate mortgage payments.* (+) b. Derive the formula for the sum of a finite geometric series.

Making Unclear Language Worse

Original Standards	Draft Revisions Reconsidered
HSA.APR.3 Understand the relationship between zeros and factors of polynomials. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Understand the relationship between zeros and factors of polynomials.** **A.APR.3** Identify zeros of polynomials when suitable factorizations are provided, and use the zeros to construct a rough graph of the function defined by the polynomial.

Overlooking Symmetry Already in High School

Original Standards	Draft Revisions Reconsidered
Draw and identify lines and angles, and classify shapes by properties of their lines and angles. **4.G.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Deleted standard.
New standard.	**Classify and analyze geometric figures.** **G.CO.15** Understand and use the concept of symmetry. **a.** Identify line-symmetric figures; draw and use lines of symmetry to analyze properties of shapes. **b.** Identify rotationally-symmetric figures; determine the angle of rotation; and use rotational symmetry to analyze properties of shapes.
Experiment with transformations in the plane. **G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	No change.

Confusing Methods of Proof

Original Standards	Draft Revisions Reconsidered
8.G.6 Understand and apply the Pythagorean Theorem. Explain a proof of the Pythagorean Theorem and its converse.	**Understand and apply the Pythagorean Theorem.** **8.G.6** Analyze and justify an informal proof (for example, pictorially, narratively, geometrically, etc.) of the Pythagorean Theorem and its converse.
HSG.CO.9 Prove geometric theorems. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	**Prove geometric theorems. Apply multiple proof methods, such as narrative paragraphs, flow diagrams, coordinate proofs, two-column proofs, diagrams without words, and the use of dynamic software.** **G.CO.9** Prove and apply theorems about lines and angles. *Theorems include but are not restricted to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

Weakening Expectations for Proof

Original Standards	Draft Revisions Reconsidered
HSA.APR.4 Use polynomial identities to solve problems. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	**Use polynomial identities to solve problems.** **A.APR.4** Verify polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*
HSF.TF.8 Prove and apply trigonometric identities. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	**Verify[G] and apply trigonometric identities.** **F.TF.8** Verify[G] the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
HSF.TF.9 Prove and apply trigonometric identities. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	**Verify[G] and apply trigonometric identities.** (+) **F.TF.9** Verify[G] the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
HSG.C.1 Understand and apply theorems about circles. Prove that all circles are similar.	**Understand and apply theorems about circles.** **G.C.1** Verify that all circles are similar.
HSG.C.3 Understand and apply theorems about circles. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	**Understand and apply theorems about circles.** **G.C.3** Construct the inscribed and circumscribed circles of a triangle; verify[G] and apply properties of angles for a quadrilateral inscribed in a circle.

Widening High School

Original Standards	Draft Revisions Reconsidered
HSG.SRT.10 Apply trigonometry to general triangles. (+) Prove the Laws of Sines and Cosines and use them to solve problems.	**Apply trigonometry to general triangles.** **G.SRT.10** Explore proofs of the Laws of Sines and Cosines and use the Laws to solve problems.
HSG.SRT.11 Apply trigonometry to general triangles. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	**Apply trigonometry to general triangles.** **G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.
HSG.SRT.9 Apply trigonometry to general triangles. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	**Apply trigonometry to general triangles.** **G.SRT.9** Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
HSS.CP.8 Use the rules of probability to compute probabilities of compound events in a uniform probability model. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.** **S.CP.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.*
HSS.CP.9 Use the rules of probability to compute probabilities of compound events in a uniform probability model. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.** **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.*

Key Features and Challenges

- Checks and balances
- Getting the right people involved
- Higher education involvement
- Encouraging learning through the standards

Continuing Needs

- More focus in the high school
- More polish in the geometry progression
- Support for proof, in more than geometry
- Support for career-intending students